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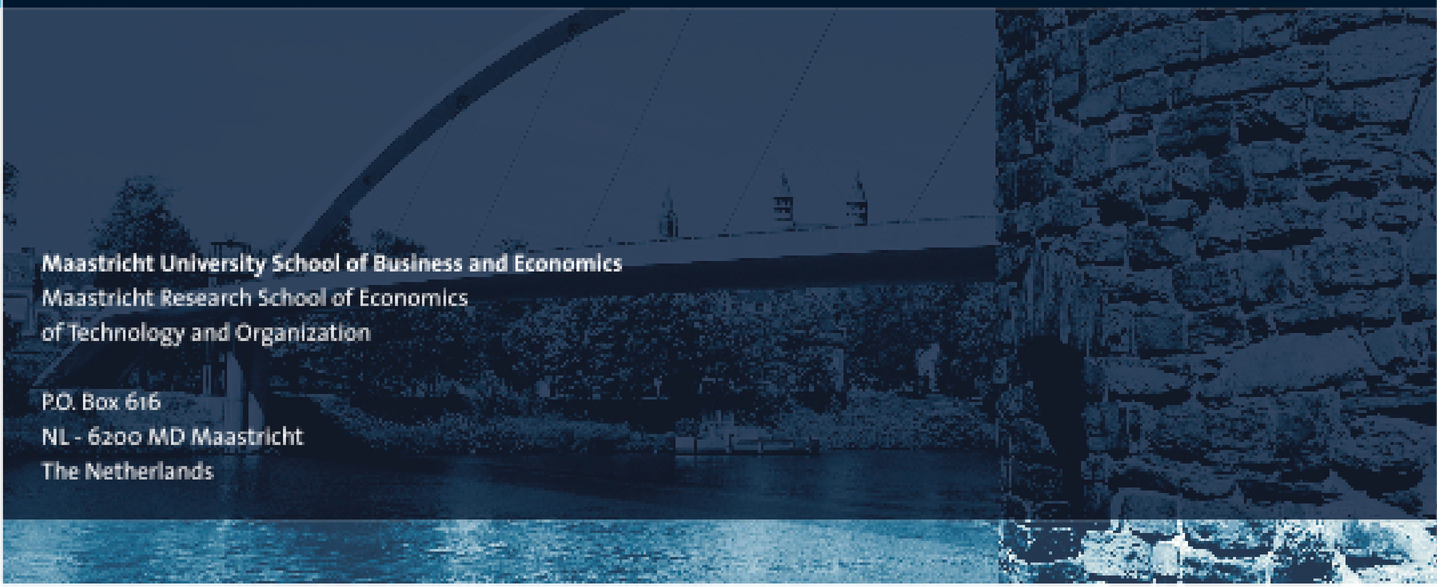

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Investigating the Zero Lower Bound on the Nominal Interest Rate under Financial Instability*

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Abstract

This paper introduces a zero lower bound constraint on the nominal interest rate in a financial accelerator model with nominal and real rigidities. We first analyze the implications for aggregate dynamics of binding the zero lower bound for shocks that depress the nominal interest rate. We include a sudden decrease in the value of the business sector net worth and an increase in its returns volatility, as two financial shocks that originate in the endogenous credit market of the model. We then explore the effects of the central bank management of expectations and a fiscal stimulus in a deep recession scenario, where the interest rate initially binds its zero bound. We find that a commitment by the central bank to keep the interest rate low for more time than prescribed by a typical interest rate rule may indeed reduce the volatility of output and inflation. For government purchases, we find a fiscal multiplier greater than one for at least 5 quarters. This is due to the presence of the zero lower bound and the Fisher (1933)'s debt-deflation channel, which implies that government spending may reduce the business sector risk premium and thus the cost of investment.

Keywords: Zero Lower Bound, Financial Accelerator Model, Financial Shocks, Government Spending, Public Expectations

JEL codes: E31; E44; E52; E58

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1 Introduction

Recent economic developments have renewed the interest in the analysis of the zero lower bound on nominal interest rates. On the one hand, the responses to the 2007-2008 financial crisis by different central banks have included the reduction of their target interest rates to record low levels. In the United States, for instance, the short-term interest rate has been set to a range of 0 and 0.25 percent since December 2008. On the other hand, the financial turmoil is a reminder to macroeconomic researchers about the impact that financial shocks may have to the rest of the economy. In a framework of financial instability, one may ask what is the efficiency of different economic policies, such as a counter-cyclical fiscal stimulus or the management of expectations by the central bank, in rebounding the economy from a deep recession. The present paper aims to shed some light on this question.

We use a financial accelerator model calibrated to the U.S. with nominal and real rigidities plus a zero lower bound constraint on the nominal interest rate to pursue our analysis. The so-called financial accelerator framework *à la* Bernanke, Gertler, and Gilchrist (2000, BGG hereafter) has been extensively used in the literature due to its ability to amplify business cycles and reproduce the observed counter-cyclical business sector risk premium.[1] The model features financial frictions due to the presence of moral hazard in the credit market, and its advantages rely on two key factors. First, it introduces a balance sheet channel which influences the dynamics of the external finance premium and thus the cost of investment. This transmission device is very similar to Fisher (1933)'s discussion about the debt-deflation channel that collaborated to the worsening of economic conditions during the Great Depression. Thus, the financial accelerator model contains richer dynamics than a standard New Keynesian model.[2] Second, including a financial sector allows us to study shocks that originate in the endogenous credit market of the model, along with their spillover effects to rest of the economy.

[1] The financial accelerator model *à la* BGG have been extensively employed to emphasize the amplification effects of financial frictions (Bernanke *et al.*, 2000, Gilchrist and Leahy 2002, Meier and Müller, 2006, Faia, 2007, Christensen and Dib, 2008, among many others). More recently, some authors have investigated the relative importance of financial shocks over the business cycle by estimating the financial accelerator model (Christiano *et al.*, 2003-2009b, Fuentes-Albero, 2009, Gilchrist *et al.*, 2009, Nolan and Thoenissen, 2009, Queijo von Heideken, 2009).

[2] There are few examples of how the financial accelerator model has been used to understand better the fluctuations of aggregate variables. For instance, Christiano *et al.* (2003) add the financial accelerator mechanism to a state-of-the-art New Keynesian model with nominal rigidities, and use their framework to study the Great depression in the U.S. More recently, Gertler *et al.* (2007) show that a calibrated version of the financial accelerator model captures well the behavior of the Korean economy during the Asian financial crisis of 97-98. Finally, Queijo von Heideken (2009) and Christiano *et al.* (2009b) estimate the financial accelerator model for the U.S. and Europe, and conclude that aggregate data is better explained when the financial accelerator is present.

Using this framework, we first analyze the qualitatively effects on aggregate dynamics of shocks on technology, preferences, and financial markets that induce the nominal interest rate to hit its zero lower bound. The financial shocks we consider are a sudden decrease in the value of the borrowers' collateral (net worth) and an increase in the borrowers' returns volatility. We assume that these shocks are sufficiently big to force the nominal interest rate to hit its zero lower bound at impact. After characterizing the economic dynamics under the zero lower bound constraint, we explore economic policies that search to steer the economy back to the steady-state in a deep recession scenario. The economic measures we consider are a fiscal stimulus, in terms of an increase in government purchases, and the management of public expectations, in terms of the future path of the nominal interest rate, as recommended by Eggertsson and Woodford (2003) or Jung *et al.* (2005). The hypothetical recession is generated from a negative net worth shock and a shift from spending to saving by households (i.e., a preference shock). We choose to combine these shocks in order to increase the probability that the interest rate hit its zero bound and, thus, analyze a relevant exercise.[3]

A number of authors have investigated the consequences of having a zero nominal interest rate in the economy. Reifschneider and Williams (2000), McCallum (2000), Coenen and Wieland (2003), Coenen *et al.* (2004), Wolman (2005), Dieppe and McAdam (2006), and Bodenstein *et al.* (2009) investigate the performance of different policy targets and monetary rules at reducing the volatility of macroeconomic dynamics in the presence of the zero floor of nominal interest rates. The role of monetary policy strategies under the zero bound has also been investigated through optimal monetary policy analysis by Krugman (1998), Eggertson and Woodford (2003), Jung *et al.* (2005), Adam and Billi (2006), Nakov (2008), Oda and Nagahata (2008), Levin *et al.* (2009), Walsh (2009), among others. One of the most prominent recommendations of this literature is the use of *forward guidance* in terms of the future path of the nominal interest rate. The latter requires a commitment from the central bank to announce and keep the interest rate at low levels even when the economy shows signs of recovery. This provides a stimulus to the economy if it lowers the expected future real interest rate of consumers and investors, i.e., if inflation expectations are successfully lifted. If so, these agents would start to smooth consumption and investment from today, making the recession milder. We are interested to see the effects of this policy in the context of a deep recession using the financial accelerator model. We evaluate this policy in terms of the volatility of inflation and

[3] Reifschneider and Williams (2000), Schmitt-Grohe and Uribe (2007), Amano and Shukayev (2009), among others, argue that in order to bind the zero lower bound constraint, single shocks would have to be quite big with respect to what is usually estimated. Thus, the probability that the zero lower bound constraint binds with a single shock is very low.

output that would result with and without its implementation. At least to our knowledge, this has not been done using a model displaying financial frictions.

When in turns to fiscal policy, there is an ongoing discussion about the effectiveness of government spending at stimulating output. The discussion has been focus on the size of the government purchases multiplier. On the one hand, there is an unsatisfactory evidence in favor of fiscal multipliers greater than one, either on the theoretical or empirical grounds (see Auerbach and Gale, 2009). However, few authors argue that certain conditions may induce the appearance of fiscal multipliers greater than unity. For instance. Christiano et *al.* (2009a) argue that the multiplier can be large when the nominal interest rate of the economy is equal to zero for some time. Romer and Bernstein (2009), in a document that argued in favor of the 2009 *American Recovery and Reinvestment Act*, find a multiplier of 1.6 using a model where, apparently, the interest rate is equal to zero at all times. Cogan et *al.* (2010), using the Smets and Wouters (2007) model, find that the fiscal multiplier would be just slightly greater than 1 at impact and smaller afterwards, even if the zero bound is binding for some time. These authors, however, use model environments in which the balance sheet channel is not present and thus they do not analyze the impact of government spending on the external finance premium and investment. Fernández-Villaverde (2010) uses a financial accelerator model to show that the fiscal multiplier is nearly 1 at impact, but does not investigates the impact of fixed interest rates on the multiplier. The contribution of our paper on this respect is to stress the role of the balance sheet channel on amplifying the effects of the fiscal stimulus, in a situation where there are financial frictions and the nominal interest rate is equal to zero for some time.

We obtain several results. First, we show that the economy features more volatility after a shock that induces the nominal interest rate to hit its zero bound. This expected result comes from the lost of the interest rate as the monetary policy instrument. Second, we find that if the central bank commits to keep the interest rate at lower levels for more time than prescribed by a typical interest rate rule, it may indeed reduce the volatility of output and inflation. There is, however, an optimal commitment period, from which beyond this point the volatility of output and inflation increases. Third, we find that an increase in government purchases has a fiscal multiplier greater than one for at least five quarters when the economy suffers from an important decrease in the business sector net worth and a shift from spending to saving. This is not only due to the presence of the zero lower bound for a sizable amount of time, like in Christiano *et al.* (2009a), but also to the impact that the government spending has on the external finance premium and investment, which originates from the balance sheet channel of the model (as in Fernández-Villaverde 2010). And fourth, the fiscal multiplier is greater than one for a long period of time (i.e. almost two years) when a policy mix between fiscal policy and forward guidance is implemented. That is, the zero lower bound on the nominal interest rate increases the net effect of government spending as a counter-cyclical policy.

The rest of the paper is as follows. Section 2 presents recent economic developments in the U.S., and make a connection to the environment assumed by the financial accelerator framework. Section 3 describes the model while Section 4 elaborates on its calibration and explains the solution method when the interest rate is equal to zero. Section 5 characterizes the responses of certain macroeconomic variables to different shocks that depress the interest rate. Section 6 reviews the effects of fiscal policy and management of public expectations in an economy hit by negative net worth and preference shocks. Finally, some concluding remarks are offered.

2 Recent Economic and Business Sector Developments in the U.S.

Figure 1 displays the evolution of certain financial and economic variables in the U.S. from the start of the Great Moderation in 1986 up to the end of 2009. We find convenient to present this data since it might give us a sense of the qualitative predictions of the model, and because the latter is calibrated for the case of the U.S. The financial data corresponds to the sum of corporate and noncorporate businesses, in the nonfarm nonfinancial sector, provided by the Flow of Funds account of the Federal Reserve (tables B.102 and B.103). All financial variables are in real and per capita terms.

Real estate assets are a component of tangible assets. Panel A of Figure 1 shows how the value of real estate assets determines a great deal of the trend of tangible assets. Financial assets and tangible assets add up to total assets in the balance sheet of the nonfarm and nonfinancial business sector. The first panel also shows how financial assets increase at a great speed at the beginning of the new millennium, mainly due to financial innovation.

The more striking feature in the balance sheet of the nonfinancial business sector is given by the substantial increase in the value of real estate assets from 2004:IV to 2007:II. This mainly reflected the boom in the housing market that occurred during those years. In contrast, panel B shows that total liabilities and debt, measured as the value of credit market instruments, did not follow the rise of total assets in the same period, giving as a consequence an important increase in the total net worth of the business sector (i.e. total assets minus liabilities). This is shown in panel C of Figure 1. Fuentes-Albero (2009) considers that a financial accelerator model *à la* BGG may explain better a measure of the net worth composed by the difference between tangible assets and credit market instruments. This is so because the BGG model does not capture the dynamics of financial assets, nor the distortionary effects of business taxes. Panel C also shows the proposed measured of net worth according to Fuentes-Albero, denoted as NW.Alt. The latter is characterized by a higher stability than the total net worth up to the start of the real estate boom in 2004.

A higher growth of tangible assets in comparison with the growth of debt during the housing bubble resulted in a lower leverage ratio for the same period. Panel D shows a consistent measure of the leverage ratio in terms of the tangible-assets-to-NW.Alt ratio. Other measures of leverage, such as the debt-to-equity ratio, or the total-assets-to-total-net worth ratio display similar trends. Panel D also presents a measure of the market perceived risk premia of the business sector, which is measured as the spread between the Baa corporate bonds rate and the 10-year Treasury bill yield. Gilchrist *et al.* (2009) argue that different measures of the corporate bonds spread may represent well the unobserved external finance premium that is referred in the BGG model, and which is closely affected by the balance sheet of the business sector. Notably, panel D shows that the risk premia diminished during the real estate boom, following the decrease in the business sector leverage ratio. This is consistent with the financial accelerator model, which predicts that a firm's default probability is positively affected by the firm's leverage ratio. As such, if the latter decreases, the market will set a lower risk premia which reflects also a lower probability of default.

From the second half of 2007, the value of real estate assets start to decrease aggressively along with the aggregate business sector net worth. As a consequence, the leverage ratio and the Baa Corporate bond spread soared to levels not registered at least in the last 50 years. The impact of this important financial disruption on economic activity can be seen in panels E to H of Figure 1. The output gap and unemployment gap, both measured as percent deviation from the Congressional Budget Office's estimations of potential output and the natural rate of unemployment, have substantially widened.[4] Inflation, measured as the percent change of the GDP deflator with respect to the same quarter one year ago, has also fallen substantially, and is well bellow the implicit target of 2 percent. As a response, the Federal Reserve started to decrease the fed funds rate from nearly above 5 per cent in the second quarter of 2007, down to 0 per cent by end of 2008, as shown in panel G. The effective fed funds rate has been between 0 and 0.25 per cent since then, and there is so far not a clear signal that it will be raised in the near future. Finally, panel H shows the evolution of the University of Michigan's consumer sentiment index, which measures the households' optimism about the state of the economy and may reflect changes in savings and spending activity. Important drop of this index may be interpreted as less

[4]The CBO potential output is taken from its January 2009 estimation.

willingness to spend by households. Consumer sentiment accumulated a substantial drop from the start of 2007 to the end of 2008, falling from 92 points to 57.

These are by far the major disruptions since the start of the Great Moderation. The drop in the business sector net worth and a higher risk premia imply that the capacity to invest of firms has been substantially diminished. On the other hand, consumer sentiment also shows that households intended to decreases their spending. One shall not forget the role of banks in the deterioration of credit conditions, which added to the worsening of aggregate demand. Banks and other financial institutions resulted severely affected in their balance sheets by the implosion of the subprime mortgage market and its related financial derivatives. However, the BGG framework does not incorporate an interbank market, and thus it remains silent about the solvency, liquidity, and asymmetric information problems that eroded from this market as well.

3 The Model

The framework is based on the workhorse New-Keynesian models, including real and nominal frictions, and enriched with frictions on the credit market (Bernanke *et al.*, 2000).The model is composed of households, firms, entrepreneurs, capital producers, financial intermediaries, a central bank and a government.

Frictions in the credit sector arise from asymmetric information between entrepreneurs (borrowers) and the representative financial intermediary (lender): the lender pays a monitoring cost to observe the individual defaulted entrepreneur's realized return, while borrowers observe it for free. This results in an increasing relationship between the external finance premium and the entrepreneurs' net worth. During a financial downturn, asset prices fall and balance sheet of entrepreneurs deteriorates, leading to an increase in the external finance premium which in turn discourages investment. This amplification effect of shocks on the activity corresponds to the financial accelerator mechanism. The model also features the so-called "debt-deflation Fisher effect" since the debt contracts are denominated in terms of the nominal interest rate. An unexpected decrease in inflation, driven by a economic downturn, deteriorates the net worth of entrepreneurs and reinforce the decline in the activity.

Following the New Keynesian literature, the model features several real and nominal rigidities which play a key role to ensure a good match between the model's predictions and the data. We assume that households form habits in their consumption pattern, prices and wages are set according to the staggered-price setting *à la* Calvo (1983) and they are indexed on inflation. In the production sector, capital utilization rate is variable and the representative capital producer pays investment adjustment cost. Finally, our model differs from the widespread monetary DSGE model in one important way: central bank's decisions are leaded by the Taylor rule but in case of strong negative shock, the monetary authorities face the zero lower bound constraint in the nominal interest rate.

3.1 Households

Preferences The economy is inhabited by a continuum of differentiated households, indexed by $i \in [0,1]$. A typical household selects a sequence of consumption, wages, and savings that are invested in a financial intermediary that pays the riskless rate of return. Households differ by the specific labor type they are endowed with, which gives them monopolistic power to set their own wage. Household i's objective is to maximize her expected sequence of present and future utility flows given by

$$\mathrm{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \varepsilon_t \mathbb{U}(c_T - bc_{T-1}) - \mathbb{V}(\ell^h_{i,T}) \right\},$$

subject to the sequence of constraints

$$c_t + \frac{d_t}{R_t} \leq w_{i,t}\ell^h_{i,t} + \frac{d_{t-1}}{1+\pi_t} + \frac{\Upsilon_t}{P_t} + \mathrm{div}_t, \tag{1}$$

where E_t is the expectation operator conditional to the information available in period t. $\beta \in (0,1)$ is the subjective discount factor and $b \in [0,1)$ is the habit parameter; c_t denotes real consumption; P_t is the price of final goods; $w_{i,t} \equiv W_{i,t}/P_t$ and $\ell^h_{i,t}$ denote the real wage and the labor supply of type-i household's at period t; $1+\pi_t = P_t/P_{t-1}$ represents the gross inflation rate; $d_t \equiv D_t/P_t$, where D_t denotes the nominal deposits held at the financial intermediary in period t and maturing in period $t+1$; R_t denotes the riskless gross nominal interest rate; div_t denotes real profits redistributed by monopolistic firms; and Υ_t is a nominal transfer from the government. In addition, ε_t denotes a preference shock which follows an autorregressive process of the form

$$\log(\varepsilon_t) = \rho_\varepsilon \log(\varepsilon_{t-1}) + \epsilon_{\varepsilon,t},$$

where $\rho_\varepsilon \in (0,1)$, and $\epsilon_{\varepsilon,t} \sim \mathrm{iid}(0, \sigma_\varepsilon)$.

The first order conditions with respect to c_t and d_t are

$$\varepsilon_t \mathbb{U}_c(c_t - bc_{t-1}) - \beta b \mathrm{E}_t\{\varepsilon_{t+1}\mathbb{U}_c(c_{t+1} - bc_t)\} = \lambda_t. \tag{2}$$

$$\frac{\lambda_t}{R_t} = \beta \mathrm{E}_t \left\{ \frac{\lambda_{t+1}}{1+\pi_{t+1}} \right\}. \tag{3}$$

where $\mathbb{U}_c(\cdot)$ denotes the derivative of $\mathbb{U}(\cdot)$ w.r.t. c_t, and λ_t is the Lagrangian multiplier associated to the budget constraint. Equation (2) defines the marginal utility of consumption. Equation (3) is the risk free bond equation which establishes that the ratio of the marginal utility of future and current consumption is equal to the inverse of the real interest rate.

Wage Setting A typical household i acts as a monopoly supplier of type-i labor. Following Erceg et *al.* (2000), we assume that the set of differentiated labor inputs, indexed by $i \in [0, 1]$, are aggregated into a single labor input ℓ_t^h by a competitive labor intermediary. The latter produces the aggregate labor input according to the following CES technology

$$\ell_t^h = \left(\int_0^1 \left[\ell_{i,t}^h \right]^{(\theta_w - 1)/\theta_w} \mathrm{d}i \right)^{\theta_w/(\theta_w - 1)}, \tag{4}$$

where $\theta_w > 1$ is the elasticity of substitution between any two labor types. The associated aggregate nominal wage obeys

$$W_t = \left(\int_0^1 W_{i,t}^{1-\theta_w} \mathrm{d}i \right)^{1/(1-\theta_w)}, \tag{5}$$

where $W_{i,t}$ denotes the nominal wage rate associated to type-i labor.

Following Calvo (1983), it is assumed that at each point in time only a fraction $1 - \alpha_w$ of the households can set a new wage, which will remain fixed until the next time period the household is drawn to reset its wage. The remaining households simply revise their wages according to the rule

$$W_{i,T} = \delta_{t,T}^w W_{i,t},$$

with

$$\delta_{t,T}^w = \begin{cases} \prod_{j=t}^{T-1} (1+\pi)^{1-\gamma_w} (1+\pi_j)^{\gamma_w} & \text{if } T > t \\ 1 & \text{otherwise} \end{cases},$$

where $1 + \pi$ is the steady-state inflation rate and $\gamma_w \in (0, 1)$ measures the degree of indexation to the most recently available inflation measure. Since the household is a monopoly supplier, it internalizes the demand for its labor when setting its wage. Additionally, it takes into account the fact that this *optimal* wage rate will presumably hold for more than on period -except for the automatic revision. Now, let $W_{i,t}^\star$ denote the nominal wage rate chosen by type-i household at time t, and $\ell_{i,t,T}^{h\star}$ the hours worked in period T if type-i household last re-optimized its wage in period t. Type-i household selects $W_{i,t}^\star$ in order to maximize her expected lifetime utility. The first order condition is described by

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \ell_{i,t,T}^\star \left\{ \lambda_T \delta_{t,T}^w \frac{w_{i,t}^\star}{w_t} \frac{w_T}{1 + \pi_{t,T}^w} - \mu_w \mathbb{V}_\ell \left(\ell_{i,t,T}^{h\star} \right) \right\} = 0, \tag{6}$$

where $\mathbb{V}_\ell(\cdot)$ denotes the derivative of $\mathbb{V}(\cdot)$ w.r.t. ℓ, $w_t \equiv W_t/P_t$ denotes the aggregate real wage, $1 + \pi_{t,T}^w \equiv W_T/W_t$ denotes wage inflation, $w_{i,t}^\star \equiv W_{i,t}^\star/P_t$ is the time t optimal real wage, and $\mu_w = \theta_w/(\theta_w - 1)$ denotes the wage mark-up.

3.2 Entrepreneurs

Optimal Financial Contract There are a continuum of risk neutral entrepreneurs, indexed by $e \in [0,1]$. At time t, type-e entrepreneur purchases at price Q_t the stock of capital $\tilde{k}_{e,t+1}$ for use in $t+1$. She finances her capital expenditures by her own internal resources and debt. Let $n_{e,t+1}$ be the available real net worth of type-e entrepreneur at the end of period t and $b_{e,t+1}$ the amount of real debt asked to the financial intermediary (or lender). Accordingly,

$$q_t \tilde{k}_{e,t+1} = b_{e,t+1} + n_{e,t+1}, \tag{7}$$

where $q_t \equiv Q_t/P_t$. The lender is also risk neutral and obtains funds from households, who are promised to receive back their deposits plus interest earnings according to the riskless gross rate of return, which in real terms is given by $r_t = R_t/\mathrm{E}_t(1+\pi_{t+1})$. Following Bernanke *et al.* (2000), it is assumed that the ex-post gross return on capital for type-e entrepreneur, $r^k_{e,t+1}$, is affected by an idiosyncratic disturbance, denoted by $\omega_{e,t+1}$. The latter is an i.i.d. random variable across time and types, with a continuous and once-differentiable c.d.f., $F(\omega)$, over a non-negative support. Also, it is considered that $\omega_{e,t}$ is unknown to both the entrepreneur and the lender prior to the investment decision, with $\mathrm{E}(\omega) = 1$ and $\mathrm{V}(\omega) = \sigma^2_{\omega,t}$. In line with Christiano *et al.* (2009b), $\sigma^2_{\omega,t}$ is the realization of a stochastic process, capturing the notion that the riskiness of entrepreneurial projects varies over time. An increase in $\sigma^2_{\omega,t}$ widens the distribution of the idiosyncratic disturbance and rises heterogeneity among entrepreneurs, which affects their default probability. The volatility parameter $\sigma_{\omega,t}$ is assumed to follow the autoregressive process

$$\log(\sigma_{\omega,t}) = \rho_\sigma \log(\sigma_{\omega,t-1}) + (1-\rho_\sigma)\log(\sigma_\omega) + \epsilon_{\sigma,t},$$

where ρ_σ and $\sigma_\omega \in (0,1)$, and $\epsilon_{\sigma,t} \sim \mathrm{iid}(0,\sigma_\sigma)$. In the spirit of Townsend (1979), lenders bear a fixed monitoring cost in order to observe the borrowers' realized return, while borrowers observe it for free. This particular setting raises the problem of moral hazard, with borrowers possibly misreporting their realized returns to fake a bankruptcy. The monitoring cost is a proportion $\mu \in [0,1]$ of the realized gross payoff to the entrepreneur's capital, i.e., $\mu\omega_{e,t+1} r^k_{e,t+1} q_t \tilde{k}_{e,t+1}$.
The type-e entrepreneur chooses the value of his project's capital, $q_t \tilde{k}_{e,t+1}$, and the associated level of borrowing, $b_{e,t+1}$, prior to the realization of $\omega_{e,t+1}$. The optimal contract is characterized by a gross non-default loan rate, $r^g_{e,t+1}$, and a threshold value of the idiosyncratic shock, say $\bar{\omega}_{e,t+1}$, such as for values of $\omega_{e,t+1}$ greater or equal than $\bar{\omega}_{e,t+1}$, the entrepreneur repays its debt at rate $r^g_{e,t+1}$. Thus, $\bar{\omega}_{e,t+1}$ and $r^g_{e,t+1}$ are defined by

$$\bar{\omega}_{e,t+1} r^k_{e,t+1} q_t \tilde{k}_{e,t+1} = r^g_{e,t+1} b_{e,t+1}.$$

In the case where $\omega_{e,t+1} < \bar{\omega}_{e,t+1}$, the entrepreneur declares bankruptcy and the lender pays the monitoring cost to audit the entrepreneur, and keeps all of the borrower's realized returns. The lender participates in the contract as long as he is assured to receive an expected loan return equal to the opportunity costs of its funds. Since it is assumed that the lender can perfectly diversify the risk associated with the loan, its relevant opportunity cost is represented by the economy riskless rate r_t.

A detailed description of the financial contract is provided in the technical appendix, available upon request. Bernanke *et al.* (2000) also offer a detailed treatment of the lender problem. However, two assumptions need to be highlighted. First, since the lender and all entrepreneurs are risk neutral and households are risk averse, all systematic risk along with the expected defaulting cost is passed through the lenders and absorbed by entrepreneurs. And second, it is assumed that entrepreneurs have a linear utility in consumption and are subject to similar aggregate shocks, which allows for the aggregation of the financial contract terms, such as $r^k_{e,t+1} = r^k_{t+1}$ for all e, and $\tilde{k}_{t+1} = \int \tilde{k}_{e,t+1} de$. More importantly, the aggregation allows for an identification of a common value for the financial threshold $\bar{\omega}$, such as $\bar{\omega}_{e,t+1} = \bar{\omega}_{t+1}$for all entrepreneurs.

Let us introduce some definitions. Allow $\Gamma(\bar{\omega}_{t+1})$ and $\mu G(\bar{\omega}_{t+1})$ to describe the expected gross share of profits going to the lender, and the expected monitoring costs, respectively. These expressions are given by

$$\Gamma(\bar{\omega}_{t+1}) = \bar{\omega}_{t+1} \int_{\bar{\omega}_{t+1}}^{\infty} f(\omega) d\omega + \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega, \quad \text{and}$$

$$\mu G(\bar{\omega}_{t+1}) = \mu_t \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega.$$

Thus, the optimal lending contract consists in choosing $\tilde{k}_{t+1}$ and $\bar{\omega}_{t+1}$ in order to maximize the entrepreneurs expected returns subject to the participation constraint of the lender, i.e.

$$\max_{\tilde{k}_{t+1}, \bar{\omega}_{t+1}} \mathrm{E}_t \left[(1 - \Gamma(\bar{\omega}_{t+1}))\, r^k_{t+1} q_t \tilde{k}_{t+1} \right],$$

subject to

$$\mathrm{E}_t \left[(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}))\, r^k_{t+1} q_t \tilde{k}_{t+1} \right] \geq \mathrm{E}_t \left[r_t (q_t \tilde{k}_{t+1} - n_{t+1}) \right].$$

It is worth to notice that the optimal solutions for $\tilde{k}_{t+1}$ and $\bar{\omega}_{t+1}$ are actually state-contingent functions that depend on the realized values of r^k_{t+1}, since the latter is subject to the aggregate uncertainty related to the general equilibrium model. Let $\check{r}_t = \mathrm{E}_t \{ r^k_{t+1} / r_t \}$ be the expected discounted return on capital. The first order conditions of the above problem imply that, in equilibrium, the discounted return on capital will be equal to the marginal cost of external finance, i.e.

$$\check{r}_t = x \left(\frac{n_{t+1}}{q_t \tilde{k}_{t+1}} \right), \tag{8}$$

with $x'(\cdot) < 0$, for $n_{t+1} < q_t\tilde{k}_{t+1}$. The intuition behind function $x(\cdot)$ is quite simple: All else equal, the cost of external finance should increase whenever the net worth to capital investment ratio decreases. This is so, since at the same time the entrepreneurs' debt is increasing. This, in turn, increases the probability of default and requires the cost of borrowing higher in order to meet the expected participation constraint of the lender. This is the key feature of the financial accelerator model, and allow us to treat the expected discounted return on capital, $\check{r}_t$, as the external finance premium.

Entrepreneurs in General Equilibrium Type-e entrepreneur, that owns the stock of capital $\tilde{k}_{e,t}$, provides capital services $k_{e,t}$ to intermediate firms, according to

$$k_{e,t} = u_{e,t}\tilde{k}_{e,t},$$

where $u_{e,t} > 0$ is the individual rate of capital utilization. At the beginning of period t, after observing all the shocks, entrepreneurs choose how intensively to use their capital. They rent capital services to intermediate firms, and once goods have been produced, they sell to the capital producer the remaining un-depreciated stock of capital. Thus, the gross return to holding a unit of capital from $t-1$ to t can be written as

$$r^k_{e,t} \equiv \frac{u_{e,t}z_t + (1-\delta(u_{e,t}))q_t}{q_{t-1}}. \tag{9}$$

where z_t is the real payment for capital services, and $\delta(u) \in [0,1]$ is a convex depreciation function around the steady-state. Similar to Queijo von Heideken (2009), we consider a function with $\delta(0) = 0$, $\lim_{u\to\infty}\delta(u) = 1$, and in the steady-state $\delta(1) = \delta$. Entrepreneurs choose the rate of capital utilization by maximizing Equation (9) with respect to $u_{e,t}$. Notice that, since z_t and q_t are aggregate prices, all entrepreneurs will choose exactly the same rate of capital utilization, independently of their own capital holdings. Thus,

$$r^k_{e,t} = r^k_t \text{ and } u_{e,t} = u_t \quad \forall\ e,$$

such as

$$z_t = \delta'(u_t)q_t.$$

Following Bernanke *et al.* (2000) and Carlstrom and Fuerst (1997), entrepreneurs participate in the general labor market by supplying one unit of labor every period, earning the nominal wage W^e_t. Finally, each entrepreneur has a random probability of exit the economy of $1-\gamma_t$. This captures the idea that entrepreneurs cannot accumulate enough wealth to be fully self-financed. A decrease in the value of γ_t reduces the aggregate net worth, since more entrepreneurs leave the economy. It is assumed, though, that the rate of birth of entrepreneurs equals its mortality rate, in order to keep constant the number of entrepreneurs. The aggregate net worth falls in any case, because new entrepreneurs begin with a zero net worth. We assume that the parameter γ_t follows the process

$$\log(\gamma_t) = \rho_\gamma \log(\gamma_{t-1}) + (1-\rho_\gamma)\log(\gamma) + \epsilon_{\gamma,t},$$

where $\rho_\gamma \in (0,1)$, and $\epsilon_{\gamma,t} \sim \text{iid}(0,\sigma_\gamma)$.
The aggregate real net worth of entrepreneurs at the end of period t, n_{t+1}, is given by

$$n_{t+1} = \gamma_t v_t + w_t^e \tag{10}$$

where w_t^e denotes the real wage earned by entrepreneurs and v_t equals gross revenues from capital holdings from $t-1$ to t less borrowing repayments (i.e. the entrepreneurs' equity)

$$v_t = r_t^k q_{t-1}\tilde{k}_t\left(1-\mu G(\bar{\omega}_t)\right) - r_{t-1}(q_{t-1}\tilde{k}_t - n_t) \tag{11}$$

Entrepreneurs that fail in t, consume the residual net worth such as $c_t^e = (1-\gamma_t)\varrho v_t$, where the complementary fraction $(1-\varrho)$ is transferred, in lump-sum taxes to households.

3.3 Capital Producer

At the end of period t, a competitive capital producer combines the existing capital stock of period t with final goods, which denotes aggregate investment i_t, in order to produce the stock of capital to be used in period $t+1$, i.e. $\tilde{k}_{t+1}$. The capital producer buy the available capital stock at the end of each period, and he sell it back, after producing the new capital, to the entrepreneurs at price Q_t. Following Bernanke *et al.* (2000), we assume that there are increasing marginal adjustment costs in the production of capital, so investment expenditures, i_t, yield $\Phi(i_t/\tilde{k}_t)\tilde{k}_t$ units of new capital goods, where $\Phi(\cdot)$ is increasing and concave and $\Phi(0)=0$. The problem of the capital producer is as follows

$$\max_{i_t} \quad Q_t\left[(1-\delta(u_t))\tilde{k}_t + \left[1-\Phi\left(\frac{i_t}{i_{t-1}}\right)\right]i_t\right] - Q_t(1-\delta(u_t))\tilde{k}_t - P_t i_t.$$

The inclusion of adjustment costs allow for a variable price of capital, which in turn will contribute to the volatility in entrepreneurial net worth. In equilibrium, the relative price of capital, $q_t \equiv Q_t/P_t$, is given by

$$q_t = \left[1-\Phi\left(\frac{i_t}{i_{t-1}}\right) - \Phi'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right]^{-1}, \tag{12}$$

and the aggregate capital stock evolves according to

$$\tilde{k}_{t+1} = \left(1-\delta(u_t)\right)\tilde{k}_t + \Phi\left(\frac{i_t}{i_{t-1}}\right)i_t. \tag{13}$$

3.4 Final Goods Producers

The final good, y_t, used for consumption and investment, is produced in a competitive market by combining a continuum of intermediate goods indexed by $j \in [0,1]$, via the CES production function

$$y_t = \left(\int_0^1 y_{j,t}^{\frac{\theta_p - 1}{\theta_p}} \mathrm{d}j \right)^{\frac{\theta_p}{\theta_p - 1}}, \tag{14}$$

where $y_{j,t}$ denotes the overall demand addressed to the producer of intermediate good j and θ_p is the elasticity of demand for a producer of intermediate good. The maximization of profits yields typical demand functions

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta_p} y_t, \tag{15}$$

with

$$P_t = \left(\int_0^1 P_{j,t}^{1-\theta_p} \mathrm{d}j \right)^{\frac{1}{1-\theta_p}}, \tag{16}$$

where $P_{j,t}$ denotes the price of intermediate good produced by firm j.

3.5 Intermediate Good Sector

Production Function Type-j intermediate firms produce differentiated goods by assembling services of labor and capital, namely $\ell_{j,t}$ and $k_{j,t}$, respectively. Capital services are rented from the entrepreneur which owns the capital stock. Type-j firm's total labor input, $\ell_{j,t}$ is composed by household labor, $\ell^h_{j,t}$, and entrepreneurial labor, $\ell^e_{j,t}$, according to

$$\ell_{j,t} = [\ell^h_{j,t}]^{\Omega} [\ell^e_{j,t}]^{1-\Omega}.$$

Type-j intermediate good is produced with the following constant return to scale technology

$$y_{j,t} = A_t \ell_{j,t}^{1-\alpha} k_{j,t}^{\alpha}, \tag{17}$$

where A_t is a total factor productivity shifter common to all firms, which evolves according to the following process

$$\log(A_t) = \rho_A \log(A_{t-1}) + (1 - \rho_A) \log(A) + \epsilon_{A,t},$$

where $\rho_A \in (0,1)$, $\epsilon_{A,t} \sim \mathrm{iid}(0, \sigma_A)$. Each monopolistic firm chooses capital and labor services in order to minimize real cost subject to the production function (17), taking w_t, w^e_t and z_t as given. Accordingly, labor and capital demands satisfy

$$w_t = s_t \Omega (1 - \alpha) \frac{y_t}{\ell_{j,t}}, \tag{18}$$

$$w_t^e = s_t(1-\Omega)(1-\alpha)\frac{y_t}{\ell_{j,t}^e}, \tag{19}$$

$$z_t = s_t\alpha\frac{y_t}{k_{j,t}}. \tag{20}$$

where s_t is the the real marginal cost associated to capital-labor inputs.

Price Setting In each period of time, type-j monopolistic firm's price setting decision is modeled through the Calvo's (1983) staggering mechanism. In each period, a firm faces a constant probability, $1-\alpha_p$, of being able to re-optimize its price. Firm j takes the demand function (15) into account when setting its price. Additionally, it takes into account the fact that this price rate will presumably hold for more than one period. If the firm cannot re-optimize its price, the latter is re-scaled according to the simple revision rule

$$P_{j,T} = \delta_{t,T}^p P_{j,t},$$

with

$$\delta_{t,T}^p = \begin{cases} \prod_{j=t}^{T-1}(1+\pi)^{1-\gamma_p}(1+\pi_j)^{\gamma_p} & \text{if } \; T>t \\ 1 & \text{otherwise} \end{cases},$$

where $\gamma_p \in (0,1)$ measures the degree of indexation to the most recently available inflation measure. Let denote $P_{j,t}^\star$, the nominal price chosen in time t and $y_{j,t,T}^\star$, the demand for good j in period T if firm j last reoptimized its price in period t. Therefore, firm j selects $P_{j,t}^\star$ so as to maximize the present discounted sum of profit streams. The first order condition is given by

$$\mathrm{E}_t\sum_{T=t}^{\infty}(\beta\alpha_p)^{T-t}\lambda_T\frac{y_{j,t,T}^\star}{P_{j,t}^\star}\left\{\frac{\delta_{t,T}^p p_{j,t}^\star}{1+\pi_{t,T}} - \mu_p s_t\right\} = 0, \tag{21}$$

where $p_{j,t}^\star \equiv P_{j,t}^\star/P^\star$, $1+\pi_{t,T} \equiv P_T/P_t$ and $\mu_p \equiv \theta_p/(\theta_p-1)$ denotes the mark-up of the monopolistic firm.

3.6 Resource Constraint

The production of the final good is allocated to investment, total private consumption by households and entrepreneurs, public spending, and to monitoring costs paid by lenders

$$y_t = i_t + c_t + c_t^e + g_t + \mu G(\bar{\omega}_t)r_t^k q_{t-1}\tilde{k}_t,$$

where g_t denotes government expenditures, which evolves according to the following process

$$\log(g_t) = \rho_g\log(g_{t-1}) + (1-\rho_g)\log(g) + \epsilon_{g,t},$$

where $\rho_g \in (0,1)$, $\epsilon_{g,t} \sim \text{iid}(0,\sigma_A)$. For a sake of convenience, we assume that government spending are financed with lump-sum taxes.

3.7 Monetary Policy

The nominal interest rate follows a Taylor rule whenever such rule prescribes a non-negative level for the central bank's target interest rate. If this is not the case, then the central bank simply fixes its target rate equal to zero. Following Reifschneider and Williams (2000) and Bodenstein *et al.* (2009), we introduce the concept of a notional nominal interest rate, R_t^{not} in gross terms, which is subject to the following rule

$$\frac{R_t^{not}}{\bar{R}} = \left(\frac{R_{t-1}^{not}}{\bar{R}}\right)^{\rho_R} \left[\left(\frac{1+\pi_t}{1+\pi}\right)^{a_p} \left(\frac{y_t}{y_t^f}\right)^{a_y}\right]^{1-\rho_R}, \tag{22}$$

where $\rho_R \in (0,1)$ denotes the interest rate smoothing parameter; a_p describes the elasticity of R_t^{not} to the deviation of inflation from its steady-state value, π; finally, a_y is the elasticity of R_t^{not} to the output gap, with y_t^f denoting the output level that would prevail in the absence of nominal rigidities. Notice that $\bar{R}$ denotes the steady-state level of the gross nominal interest rate, determined by $(1+\pi)\beta^{-1}$.

Let R_t denote the actual short-term interest rate implemented by the central bank. Thus, the latter chooses its instrument according to

$$R_t = \max\left(0, R_t^{not}\right). \tag{23}$$

3.8 Equilibrium

In the symmetric equilibrium, all entrepreneurs, households and firms are identical and make the same decisions. In addition, equilibrium on the labor market yields $\int_0^1 \ell_{j,t} \mathrm{d}j = \ell_t^h$. The symmetric equilibrium is characterized by an allocation $\{y_t,\, c_t,\, i_t,\, \ell_t,\, k_t,\, \tilde{k}_t,\, n_t\}$ and a sequence of price and co-state variables $\{\pi_t,\, r_t,\, r_t^k,\, q_t,\, \pi_t^w,\, z_t,\, \lambda_t\}$ that satisfy the optimality conditions in each sector, the monetary policy rule, and the stochastic shocks.

4 Methodology

4.1 Calibration

The model parameters are calibrated to fit the quarterly frequency. Table 1 describes the calibrated values for parameters related to households, firms, and economic authorities.

[insert Table 1 here]

The subjective discount factor, β, is set to 0.99, which entails a annual real interest rate of 4 per cent. The Frisch elasticity of labor supply, $\omega_w^{-1} \equiv \mathbb{V}_\ell/(\ell\mathbb{V}_{\ell\ell})$, is set to unity. The degree of habit consumption, b, is set to 0.63, while the inverse of the inter-temporal elasticity of substitution, σ, is set to 0.2. All these values are set by Christiano *et al.* (2009b).

Regarding production, the capital share in the intermediate sector, α, is set to 0.35; the rate of capital depreciation, δ, is equals 0.03, as in Christiano *et al.* (2009a); the investment adjustment cost, $\varkappa \equiv \Phi''(1)$, is calibrated to 5.86, following Smets and Wouters (2005); the elasticity of the utilization rate of capital, $\vartheta_u \equiv u\delta''(u)/\delta'(u)$, is calibrated to 0.31^{-1}, similar to Queijo von Heideken (2008). Concerning price setting, we assume that the elasticity of substitution between intermediate goods, θ_p, is set to 11, which implies a price mark-up of 10 per cent. Similarly, the elasticity of substitution between labor types, θ_p, is set to 21, which translates into a wage mark-up of 5 per cent. The degrees of price and wage rigidities, α_p and α_w, are set equal to 0.67 and 0.68 respectively, implying that the average durations between price or wage re-optimization are half of a year. Price and wage indexation parameters, γ_p and γ_w, are set to 0.75 and 0.70, respectively, as suggested by Christiano *et al.* (2009a). The steady-state inflation, π, is equal to 0.

In terms of monetary policy, the interest rate smoothing parameter, ρ_R, is calibrated to 0.88; the elasticity of the notional interest rate with respect to inflation, a_π, is set to 1.85; and the elasticity of the interest rate with respect to the output gap, a_y, is set to 0.313/4. These values follow once more the estimations of Christiano *et al.* (2009a).Finally, the steady-state share of government purchases in total output is calibrated to 0.19, which corresponds to the last decade historical average.

Table 2 shows calibrated values of parameters related to the financial sector, which are taken from Bernanke *et al.* (2000)'s results.

[insert Table 2 here]

The share of entrepreneurial wages in terms of income is set to 0.01, implying a value of $\Omega = 0.9846$. The steady-state share of capital investment that is financed by the entrepreneur's net worth, $x = \tilde{k}/n$, is calibrated to 2, meaning that the steady-state leverage ratio amounts to 50 per cent. The steady-state external finance premium, $\check{r} = r^k/r$, is set to $1.02^{0.25}$, corresponding to an annual risk spread of 200 basis points, equal to the sample average spread between the business prime lending rate and the three-month Treasury bill rate. Finally, the annual business failure rate, $F(\bar{\omega})$, is set to 3 per cent. It is assumed that the idiosyncratic productivity shock, ω_t, has a log-normal distribution with positive support, and an unconditional expectation equal to 1. These moments help to determine the steady-state survival probability of entrepreneurs, γ, which is set to 0.9788, the monitoring costs to realized payoffs ratio, μ, which amounts to 0.1175, the steady-state variance of the entrepreneurs' idiosyncratic shock, σ_ω, which is equal to 0.2764, and the steady-state idiosyncratic threshold is set to 0.4982.[5]

4.2 Zero Lower Bound: Solution Strategy

The zero lower bound constraint, described in Equation (23), introduces an important non-linearity into the system. Had this constraint not appeared, we could proceed to analyze the dynamics of the economy using the linear rational expectations solution that can be derived from the above system. In fact, all the model equations can be linearized except for the nominal interest rate, which imposes different dynamics depending on whether the ZLB constraint is binding or not. We follow the piecewise-linear approach described in Bodenstein, Erceg, and Guerreri (2009) to solve for the model dynamics, which is numerically equivalent to the method employed by Eggertson and Woodford (2003). In specific, we linearized all model equations around its non-stochastic steady-state, except for the monetary policy representation.[6] We then assume that an exogenous shock hits the economy and depress the nominal interest rate so that the zero lower bound is reached at period 1, and remains in place for T periods. The zero lower bound horizon T is determined by the time-T value of the notional interest rate, which must satisfy the following condition

$$R_T^{not} < 0 \leq R_{T+1}^{not}.$$

In terms of percentage deviation from the steady-state, the latter becomes $\hat{R}_T^{not} < -\bar{r} \leq \hat{R}_{T+1}^{not}$, where $\bar{r} = 1/\beta - 1$, which is the steady-state level of the nominal interest rate. According to the calibration of the preceding section, this equals 4 per cent in annual terms. The piecewise linear system is thus conformed with two different dynamic structures: First, the dynamics when the ZLB is binding and the interest rate is equal to $-\bar{r}$; and, second, the dynamics when the ZLB is not binding and the Taylor rule is operating. The second structure imposes the determinacy conditions of the whole system and is solved using the AIM implementation (see Anderson and Moore, 1985) of the Blanchard and Kahn (1980) method for linear rational expectations models. The dynamics with the ZLB are derived using backward-induction, and are deterministic, in the sense that people make their decisions knowing that in period $T+1$ the interest rate will follow again the Taylor rule path. Bodenstein *et al.* (2009) present a detailed description of this solution algorithm.

[5] In technical terms, $\bar{\omega}$, σ_ω, γ and μ are chosen so as to satisfy the following system of steady-state equations:

$$F(\bar{\omega})= 0.03/4;\ x= 1 + \Gamma_\omega(\bar{\omega})[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]/[[1 - \Gamma(\bar{\omega})][\Gamma_\omega(\bar{\omega}) - \mu G_\omega(\bar{\omega})]],$$
$$(x-1)/x\check{r}= \Gamma(\bar{\omega}) - \mu G(\bar{\omega});\ \ n= \gamma[nr^k x[1 - \mu G(\bar{\omega})] - rn(x-1)] + w^e.$$

[6] The log-linear model is described in the appendix.

5 Model's Dynamics under the Zero Lower Bound

In this section, the model's dynamics are described for different shocks that force the nominal interest rate to hit its zero lower bound. First, we analyze the consequences of standard shocks in the literature, such as a positive technology shock (A_t) and a negative preference shock (ε_t). Afterwards, we introduce two financial shocks that put the economy into a recession: a decrease in the entrepreneurs survival probability (γ_t) and an increase in the entrepreneurs' returns volatility ($\sigma_{\omega,t}$). A fall in γ_t can be interpreted as an exogenous decrease in the entrepreneur's net worth value, as Christensen and Dib (2009), or Christiano *et al.* (2009) interpret. This shock directly deals with a shift in the demand for capital through a lower aggregate purchasing power of entrepreneurs. An increase in $\sigma_{\omega,t}$ widens the entrepreneurs' returns distribution which eventually worsens credit conditions (the quality of entrepreneurs' projects is hardly distinguishable). This can be viewed as a risk premium shock modifying credit supply conditions.[7] The size of the shocks are deliberately chosen so the nominal interest rate hits the zero lower bound at impact.

5.1 Technology Shock

The autoregressive coefficient of the technology shock, ρ_A, is calibrated at 0.945, as suggested by Christiano *et al.* (2009). The size of the innovation is chosen to force the zero lower bound at impact. Figure 2 compares the IRF of different variables in response to this shock.

[insert Figure 2 here]

Without imposing the zero lower bound, the observed dynamics of most variables are well-known. Output and investment increase, inflation drops for 6 quarters, while the nominal interest rate also decreases. Our assumption about the size of the productivity shock implies that the nominal interest rate will remain below zero for a sizable amount of time.[8]

Turning to the financial sector, there are two factors that affect the aggregate net worth. On the one hand, the rise in productivity increases the price on capital, which in turn increases the net worth (see Equations 10 and 11). On the other hand, a higher real interest rate on impact dampens future net worth, since borrowing becomes more expensive. These two effects create a

[7] Gilchrist *et al.* (2009) and Fernández-Villaverde (2009) consider a negative shock on credit supply. They assume that the external finance premium is driven by a negative stochastic component, corresponding to deterioration in lenders' health. This shock has identical qualitative implications than a shift in entrepreneurs' returns volatility. The latter could be viewed as a degradation in lenders' abilities to supply credits due to risky projects.

[8] Although the latter seems unrealistic, it help us to understand the dynamics between an economy that is constrained by the ZLB to an economy that can move freely its nominal interest rate.

humped-shape pattern for the net worth during the first 8 quarters, accompanied by an important persistence afterwards. To understand the dynamics of the risk premium, look again at Equation 8, reproduced below:

$$\check{r}_t = x\left(\frac{n_{t+1}}{q_t\tilde{k}_{t+1}}\right), \text{ with } x'(\cdot) < 0.$$

The risk premium response at impact is small, while later it smoothly increases. This is so because capital rise persistently, and this not followed by the net worth. Put it differently, a positive technology shock rises the demand for capital and increases the amount of debt that entrepreneurs can take.[9]

When the zero lower bound constraint is imposed, the economy experiences a smaller boom. This is so because, given our assumption about the size of the shock, the central bank cannot further decrease the interest rate in order to stop inflation from falling. The interest rate stays at its lower bound for 12 periods. This results in a real interest rate higher than before, which rises the cost of borrowing, induces the net worth to fall sharply, and mechanically makes the external finance premium to jump at impact. The latter provides lower incentives to invest, which explains the small response of investment on impact. Since the nominal interest rate cannot decrease further, it cannot fuel as before the booming in output, investment, and capital. As a consequence, these variables increase by less. Notice also that the volatility of inflation increases when the zero lower bound is binding.

5.2 Preference Shock

The autoregressive coefficient of the preference shock, ρ_ε, is calibrated to 0.903, as suggested by Christiano *et al.* (2009) and the negative preference shock implies that the zero lower bound is binding for 17 quarters. Figure 3 compares the IRF of macroeconomics variables in response a negative shock on households' preferences, ε_t, when the zero lower bound is present and when the nominal interest rate is free to move.

[insert Figure 3 here]

Without imposing the zero lower bound, a decrease in the marginal utility of consumption depresses consumption and output while investment is crowded in.[10] Therefore, the monetary policy rule predicts that the interest rate has to be negative to steer the economy towards the equilibrium. Since the decrease in inflation is higher than the decrease in the nominal interest rate, the real

[9] The risk premium will be counter-cyclical for a longer period if the inflation coefficient in the Taylor rule will be larger.

[10] This result is consistent with Christiano *et al.* (2007) and Christensen and Dib (2008).

interest rate rise at impact. Combined with a lower demand for capital, the latter results in a decrease in the net worth and therefore an increase in the risk premium.

When the zero lower bound constraint is imposed, inflation is more volatile, the price of capital decreases by more, and the real interest rate rise more than before. The latter brings as consequence that the net worth falls sharply at impact, while the risk premium soars. In turn, investment is crowded out due to the poor conditions in the credit market.

5.3 Net Worth Shock

The autoregressive coefficient of the survival probability shock, ρ_γ, is calibrated to 0.561, as suggested by Christiano *et al.* (2009). The size of the shock is calibrated so as to ensure that the zero lower bound constraint binds at impact.[11] The negative deviation of γ_t from its steady-state value corresponds to an exogenous drop in the value of aggregate net worth. Christiano *et al.* (2009) and Nolan and Theonissen (2009) interpret a shock in γ_t exactly in this way. For Christiano *et al.* (2009), variations in γ_t corresponds to movements in the value of assets that are not obviously linked to movements in fundamentals. Nolan and Theonissen (2009), in turn, appeal to the former Federal Reserve chairman Alan Greenspan's remark about "irrational exuberance", concerning the stock market boom in the U.S in 1996. A drop in γ_t, then, can be viewed as a decrease in the the value of entrepreneurs' assets, which will have spill over effects on the credit market, the external finance premium, and the capital market. Figure 4 compares the IRFs of some macroeconomics variables in response to a negative shock on the survival probability, γ_t. Once more, we confront an economy facing the zero lower bound versus an economy with a flexible nominal interest rate.

[insert Figure 4 here]

Without imposing the zero lower bound constraint, and after a negative net worth shock, the model predicts that the nominal interest rate will be negative for a sizable amount of time. The decrease in demand for capital drives price of capital and the real interest rate down, that makes the drop in the net worth more severe. As a consequence, the risk premium rises depressing investment. Consumption rises in the short-run because the real interest rate increases at the beginning but then sharply falls. Nevertheless, output decreases driven by investment. Inflation follows a hump-shaped pattern, decreasing at impact and then coming back at the steady-state after 3 years.

[11] It corresponds to a sizable deviation of the entrepreneurs' probability of exit (γ_t) from its steady-state (i.e. $\sigma_\gamma = 0.4$). The size of the shock needed to bind the ZLB constraint is smaller when the shock is highly persistent. In our calibration, the autoregressive parameter is low ($\rho = 0.56$), based on Christiano *et al.*'s suggestions (2009). However, Nolan and Toening (2009) suggest a highly persistent shock (i.e. $\rho = 0.98$).

When the zero lower bound constraint is imposed, the economy shows more volatility, mainly due to the lost of the monetary policy instrument. The ZLB is binding for 16 quarters. Although this might be seen excessive, it does not look unrealistic. Japan's call rate has been between 0 and 0.50 since October 1995. For the case of the U.S., Rudebusch (2009) computes a Taylor rule that fits very well the behavior of the fed funds. Such rule anticipates that the fed funds would remain for several years at its zero lower bound, and not only some months.

In comparison with the case where the ZLB is not binding, the rise in the risk premium is more severe. This is understandable, since the price of capital and the real interest rate have higher movements at impact. Thus, the net worth fall by more than before. The even worse credit conditions than in the previous case make investment and output to fall sharply. Inflation, in turn, displays again more volatility when the zero lower bound is binding. Notice also that the nominal interest rate responds strongly after the zero lower bound period is over. This is so, because the interest rate rule tries to offset the strong increase in inflation from the recovery period.

5.4 Risk Shock

What we mean by increasing the volatility of the idiosyncratic productivity shock, ω_t, is that the distribution of this random variable widens. The autoregressive coefficient of the volatility shock, ρ_ω, is calibrated to 0.850, again following Christiano *et al.* (2009). Figure 5 compares the IRFs of this shock with and without the ZLB constraint.

[insert Figure 5 here]

Although the zero lower bound is binding for 8 quarters, surprisingly the overall dynamics of the economy are not dramatically changed in one case or the other. But why this is so? An increase in volatility make harder for the lender to distinguish whether entrepreneurs default or not. The latter translate into a rise in the lending contract threshold, $\bar{\omega}_t$, and a rise in the probability of default. As a consequence, the risk premium increases. The shock in volatility is actually a risk premium shock. The latter makes investment to drop along with the price of capital which in turn diminish both output and the net worth. As in the previous demand side shocks, inflation initially drops, following the marginal cost and output, and then subsequently increases. One of the reasons for which the two scenarios are similar is due to the fact that the presence of the zero lower bound does not strongly alter the probability of default's dynamics.[12] Thus, the dynamics of the risk premium, capital prices, and net worth are almost identical in both cases.

[12] The presence of the zero lower bound does not strongly alter dynamics of the the financial sector after a risk shock because the elasticity of the volatility of ω_t with respect to the real interest rate is small.

Another reason could be that the zero lower bound constraint binds for relatively less periods than in previous cases. In order to investigate this hypothesis, we allow the persistence of the shock to increase. Figure 6 shows once again the IRFs of the shock in volatility, this time with $\rho_\omega = 0.98$.

[insert Figure 6 here]

In this case, the ZLB binds for 15 quarters. Now the differences between output, investment, and inflation are notorious. But again, the dynamics of the risk premium, the price of capital, and the net worth are roughly similar. Our conclusion is thus similar as before concerning the dynamics of the probability of default: the latter are not strongly changed by the presence of the zero lower bound, given that the original disturbance is a shock in volatility, which can be interpreted as a risk premium shock. Smets and Wouters (2007) introduce a risk premium shock (see pp. 599, Figure 2) into their setting, which looks pretty much alike to the volatility shock displayed in Figure 5.

6 Stimulating an Economy Hit by a Net Worth Shock and Scared Consumers

The purpose of this section is to analyze the effect of two different policies that intend to offset the effects of a severe recession, in which the nominal interest rate is forced to stay at its zero bound for a sizable amount of time. The ingredients of the recession are a temporal 10 percent decrease in the entrepreneurs' survival probability (such as γ_t falls from 0.98 to 0.88), and a negative preference shock. We choose to combine these shocks for two reasons. First, the presence of the two disturbances increases the probability that the interest rate hit its zero bound, and thus allow us to assume smaller shocks than in the previous section. Schmitt-Grohe and Uribe (2007), among others, argue that in order to bind the zero lower bound constraint, single shocks would have to be quite big with respect to what is usually estimated. Thus, we can argue that, in reality, is quite improbable that the nominal interest rate would be equal to zero due to the occurrence of a single shock.

Second, a negative preference shock has an special characteristic: it reflects an increase in the desire to save by the part of households, which in turn increases the demand of bonds and puts downward pressure on consumption, output, and the nominal interest rate. Bodenstein *et al.* (2009), Christiano *et al.* (2009b), or Uhlig and Drautzburg (2010) use shocks with these characteristics to induce the zero lower bound in their analyses.[13] On the other hand, one may argue

[13] Uhlig and Drautzburg (2010) assume that there is an interest rate spread shock between the risk-free rate and the rate preceived by households. A shock of this kind has exactly the same effects than a preference shock. Not

that a shift from household spending to saving typically happens when the consumer confidence index (or consumer sentiment) drops, which is likely to happen during a severe economic downturn.

Thus, our initial scenario is an economy which suffers from a deep decrease in the business sector net worth and a shift from spending to saving. Figure 7 shows the economic dynamics of the recession, illustrated by the solid line. As it can be appreciated, the nominal interest rate binds its zero bound for 16 quarters, given that the central bank follows the monetary rule described in equations (22) and (23). The economic measures we consider is a fiscal stimulus, in terms of an increase in government purchases, and the management of public expectations, or *forward guideline,* in terms of the future path of the nominal interest rate. We start the discussion with the later.

6.1 Managing Expectations

This section is motivated by the findings of Eggertsson and Woodford (2003). In an extensive discussion, these authors explore alternative policies that the monetary authority may employ when dealing with a liquidity trap. After providing theoretical reasons of why a quantitative easing strategy might do little or nothing to steer the economy, Eggertsson and Woodford (2003) discuss the implications for the economic dynamics of managing public expectations about the future path of the interest rate. The argument, accordingly, is that economic agents make their decisions taking into account their expectations about the future policy that the central bank is likely to implement. The bottom line of their discussion is that, according to their optimal path for monetary policy, the central bank should announce and maintain the nominal interest rate at very low levels for longer time than what would be normally prescribed by an strict inflation targeting rule. The latter is explained by the fact that, when agents expect a period of abundant liquidity they have a better chance to smooth consumption and plan investment.

Hereby, we perform an heuristic evaluation of this sort of policy using a representative loss function that may represent the preferences of the central bank.[14] The alternative policy is a situation in which the central bank decides and announces to keep the nominal interest at very low levels for an extensive period of time, i.e., more quarters than recommend by its Taylor rule. That is, in the initial scenario the Taylor rule prescribes that the interest rate should be positive after 16 periods. Our aim here is to explore what happens, in terms of inflation and output volatility, when the central bank decides to keep the interest rate equal to its lower bound for more then 16 periods, in

only becasue both shocks appear in the Euler equation in a similar way, but because both of them create an increase in the desire to save while induce a drop in consumption.

[14] In the derivation of the optimal monetary policy, Yung (2005) and Oda and Nagahata (2008) interprets the managing expectations policy as the "zero interest rate commitment". We adopt here a more elementary approach since we are not interested in optimal monetary policy.

despite that the Taylor rule suggests that it should increase it. We use the following loss function in order to assess the impact of this policy

$$\sum_{t=0}^{hor} \beta^t \left(\hat{\pi}_t^2 + \lambda \left(\hat{y}_t - \hat{y}_t^f \right)^2 \right). \tag{24}$$

This is a standard loss function commonly used in the literature, which is referred to be derived from a second order Taylor expansion from the utility function of the representative household. In this numerical exercise, though, we assume that deviations of output are as important as the deviations of inflation from its steady-state value, thus $\lambda = 1$. Notice as well that the output gap is defined with respect to the level of output that would prevail in the absence of nominal rigidities, y_t^f. Finally, we restrict our attention to the first 30 realizations of inflation and output, as we set $hor = 30$.

Table 3 shows the realized values of the loss function of the initial scenario, i.e. the economic recession with the central bank following the Taylor rule, and alternative scenarios, where it is assumed that the central bank holds the interest rate equal to its lower bound for more than 16 periods. Thus, if the central bank raises the interest rate after period 16, it is actually following the prescription from its monetary rule, without announcing any possibility for future abundant liquidity. In contrast, if the central bank holds the nominal interest rate equal to zero for more than 16 periods, than it must announce in a credible way that it will allow for a future period with abundant liquidity. Alternatively, it could announce that it will maintain the opportunity cost of holding money at low levels for a longer period than what it would normally do during normal economic activity. The latter is important since it gives a signal to investors and consumers that they can start to smooth future consumption and investment from today.

[insert Table 3 here]

As it can be observed from the table, the losses from inflation and output deviations are convex with respect to the number of periods in which the interest rate is held at its lower bound. This convexity suggests that there exists an optimal commitment period for the central bank to keep the interest rate at low levels. This commitment is optimal, in the sense that it minimizes the welfare losses resulting from the adverse economic conditions, and its length is conditional on the magnitude of the economic recession and the intertemporal behavior of firms and agents. The loss function value is minimal when the central bank commits to provide cheap money for 26 periods, i.e. 10 periods more than prescribed by its Taylor rule. Beyond that point, the value of the loss function increases, implying that inflation and output become too volatile. Intuitively, this is what

we would expect to see if the central bank maintains the cost of money too low for to long, specially in terms of inflation. The bottom line of this analysis is that there exists an optimal commitment period that the central bank must consider in order to provide abundant liquidity, which is in line with the analysis of Eggertsson and Woodford (2003).
The effects of the commitment policy are shown by the dash line in Figure 8. In the initial scenario the zero lower bound is hold for 16 periods, whereas in the the alternative the central bank keeps low interest rates for 10 periods more.

[insert Figure 8 here]

As we can see from the figure, the optimal commitment period alters significantly the response of real variables. Since agents expect a low interest rate for a long period of time, they respond to the adverse economic conditions by smoothing their consumption and investment. This yields a lower drop in inflation, which in turn produce a smaller increase in the real interest rate. These two facts stop the net worth to fall deeply. On the one hand, the debt-deflation channel, highlighted by Fisher (1933), is dampened, since a smaller decrease in inflation rises by less the real debt of entrepreneurs. On the other hand, a lower increase in the real interest rate also implies that the service of the debt that entrepreneurs have to pay is lower. These two effects can be observed in equation (11), which describes the gross capital gains of entrepreneurs. As a consequence of the lesser decrease in the net worth, the external finance premium increases also by less, which makes investment in the alternative scenario cheaper than in the initial one. Thus, although the economy still experiences a recession, the response of output is less volatile than if the interest rate would have followed the Taylor rule path from quarter 16 onwards. Figure 8 not only shows the argument of Eggertsson and Woodford (2003) with clarity, but also highlights the effect that the optimal commitment policy has on the external finance premium. Finally, it is worth to emphasize that in order to obtain the gains in output, the central bank must announce and make credible that it will hold the interest rate low and steady for longer time than it normally does, given the fluctuations of output and inflation.

6.2 Fiscal Stimulus

Fiscal stimulus under economics turmoil We now turn to investigate the role of a positive government shock in response to the deep recession. We seek to highlight up to what extent can the fiscal stimulus help the economy to recover from the significant net worth and preference shocks. Figure 9 shows the initial scenario again by the solid line, while the dash line represents the economy with the temporal increase in government purchases.[15]

[insert Figure 9 here]

We assume that government spending follows an autoregressive process of order one, with an autoregressive coefficient of $\rho_g = 0.945$, as suggested by Christiano *et al.* (2009a). Since in the initial scenario the recession is particularly deep, for the sake of simplicity we also assume an important increase in government spending (with the later jumping from 19 percent of GDP, its steady state value, to 23 percent of GDP at the impact period). Our main conclusions do not depend on the size of the shock, but we find it to convenient in order to highlight the main effects of government spending. Out results are as follows.

The increase in government spending can potentially shorten the duration of the zero lower bound. In our example, government spending force the nominal interest rate to become positive after 15 periods (assuming that the central bank follows its Taylor rule). Regarding output, the fiscal stimulus effectively reduces the drop in production. However, as the effects of the stimulus fade out, output eventually converges towards its initial path in the absence of the stimulus.

Perhaps the most emblematic effect of government spending in the model is its impact on the external finance premium and the rest of the financial sector. An increase in government spending expands aggregate demand, which is reflected partly in Figure 9 by a lesser decrease of inflation with respect to the initial scenario. As noticed earlier, a lower decrease of inflation dampens the Fisher's debt-deflation channel, since the entrepreneurs' real debt increases by less, as it does as well the service of this debt in real terms. The latter has a positive effect on the real capital gains of entrepreneurs, which collaborates to the lower decrease on the net worth. This reduces the moral hazard problem in the financial sector (since the collateral is greater in the fiscal stimulus scenario), which finally induces to a smaller increase in the external finance premium. A similar explanation can be found in Fernández-Villaverde (2010). Since the interest rate is constrained by its zero lower bound, investment is cheaper under the fiscal stimulus scenario then in the initial one. Thus, investment does not fall as much when the fiscal stimulus is in place.

When it comes to consumption, Figure 9 does not show a sizable impact of government spending. On the one hand, the lesser decrease in output might help consumption also to fall by less. On the other hand, the economy is subject to the Ricardian equivalence, and thus an increase of government spending crowds out consumption. These two effects offset each other in the final response

[15] Due to technical issues, we consider that the counter-cyclical fiscal policy is implemented at the same period that the ZLB constraint is binding. Christiano *et al.* (2009b) show that the effects of the government spending shock are slightly weaken if it takes two periods for the government to begin the expenditure stimulus.

of consumption, yielding the apparent lack of changes in this variable.

An important discussion in our days concerns the effectiveness of government spending in increasing output, giving the adverse economic conditions in the aftermath of the 2007-2008 financial crisis, and the several announcements of different governments about launching significant fiscal stimuli. The debate has been focus on the size of the fiscal multiplier, and the main question is whether or not it may be greater than 1, i.e. if government spending may stimulate output in more than a one-by-one basis. Christiano et *al.* (2009a) argue that the government spending multiplier can be large when the nominal interest rate is kept constant at its zero bound for some time. Romer and Bernstein (2009), in a document that argued in favor of the 2009 *American Recovery and Reinvestment Act*, find a fiscal multiplier of 1.5, which results from the average predictions of a *leading private forecasting firm* (sic) and the Federal Reserve's FRB/US model. In contrast, Cogan et *al.* (2009) find that the fiscal multiplier would be just slightly greater than 1 at impact and smaller afterwards, even if the zero lower bound is in place for 2 years. All these works use an economic environment with no financial frictions, and thus they do not incorporate the effects of government spending on the external finance premium. Fernández-Villaverde (2010) uses a model similar to the one presented here but he allows the nominal interest rate to be flexible. He finds that the fiscal multiplier would be roughly equal to 1 at impact and smaller afterwards. The next subsection is intended to add to the debate about the possible size of the fiscal multiplier, given the economic environment described herein.

Fiscal multiplier analysis In the economy hit by negative shocks in preferences and the net worth, the net effect of fiscal policy is measured by subtracting the IRFs of the initial scenario to the IRFs of the scenario with the fiscal stimulus. Let $\hat{x}_t^0$ denote the response at time t of variable x, in terms of percent deviations from the steady state, given the initial recession scenario without fiscal stimulus. Similarly, let $\hat{x}_t^{fis}$ denote the response of the same variable with the fiscal stimulus. Thus, the net effect of government spending on x is given by $\hat{x}_t^{net} \equiv \hat{x}_t^{fis} - \hat{x}_t^0$. The quarter k net fiscal multiplier is thus given by

$$\frac{\mathrm{d}y_{t+k}}{\mathrm{d}g_t} \equiv \frac{1}{\gamma_g} \frac{\hat{y}_{t+k}^{net}}{\hat{g}_t^{net}},$$

where $\gamma_g \equiv g/y$ is the steady state government spending to GDP ratio.

Figure 10 compares the value of the net fiscal multiplier in different configurations. The benchmark configuration corresponds to the baseline economy hit by a negative net worth and preference shocks, plus the expansionary government spending shock, while the nominal interest rate binds its zero lower bound for 15 periods. This is the situation described in the last section, and its fiscal multiplier is shown in Figure 10 as . As it can be noticed from the figure, output increases by slightly more than a one-by-one basis to government spending. In fact, in the first three quarters the fiscal multiplier is larger than one, with a value of 1.19 at impact and a peak value of 1.27 in the second quarter. Two years after the stimulus, the fiscal multiplier reaches 0.79. The intuition behind this result is given by the effect of the fiscal stimulus on the external finance premium, and the fact that the nominal interest rate reacts only in period 15. With these two features, the fiscal stimulus does not crowd out investment.

One may ask if the benchmark fiscal multiplier is greater than one due to the presence of the zero lower bound. Figure 10 also tackles this point: it shows that the fiscal multiplier is lower than one when the nominal interest rate can move freely, even towards hypothetical negative levels. This is shown by the line *Fin. acc., No ZLB* in the figure. In such a case, the fiscal multiplier equals 0.87 at impact and 0.33 after two years. The decrease in the multiplier can be explained as follows. A fiscal stimulus, as any other positive shock to aggregate demand, tends to increase inflation and the nominal interest rate. If the later would be allowed to be negative during the deep economic recession, the fiscal stimulus would make the interest rate less negative. Thus, if the real interest increases due to the fiscal stimulus, investment would become more expensive, which represents the classic crowding out effect. This illustrates the claim that the fiscal multiplier tends to be larger when the zero lower bound constraint is binding (Christiano *et al.*, 2009b).

Another interesting question is what is the impact of the financial accelerator model on the effect of government spending? Figure 10 shows that the fiscal multiplier is lower when we consider an economy which does not feature financial imperfections (shown by the line *No fin. acc., ZLB 15 periods*): it is greater than one only at the very short-run (it equals 1.05 at impact while it reaches 0.45 after two years). This result can be explained by the fact that in the no financial accelerator model, the risk premium is not allowed to differ from the real interest rate. Thus, the environment is equivalent to a standard New Keynesian model with real and nominal rigidities, but without credit markets imperfections. In this case, government spending has not a relevant impact on the net worth and on the external finance premium. The Fisher debt-deflation channel is closed, and thus the potency of government spending to reduce the risk premium is nil, which also foreclosures the additional incentives to investment coming from this mechanism.

Finally, we ask what is the impact on the fiscal multiplier of raising the duration of the zero lower bound in the economy. That is, we tackle the possibility that economic authorities implement an increase of government purchases along with the announcement of keeping the future nominal interest rate equal to zero for 26 periods. This is 11 periods more of what the central bank's Taylor rule would normally recommend. As expected, Figure 10 shows that the net value of the fiscal multiplier in this situation (shown by the line *Fin. acc., ZLB 26 periods*) is greater under this policy mix. In particular, the multiplier equals 1.32 at impact and 1.00 after two years, with a peak value of 1.5 in the third quarter. As explained few pages above, when agents expect a low interest rate for a longer period of time, they smooth consumption and investment, which lessens the decrease in inflation and makes the recession milder.

To conclude, these results suggest that the efficiency of government purchases to increase output in more than a one-by-one basis relies importantly on the presence of the zero lower bound and the Fisher debt-deflation channel, implied by the economy's financial frictions. The results also tell us that a policy mix is more effective at increasing output. However, it is likely that the central bank's optimal commitment period to keep the interest rate low is affected by the fiscal stimulus. One would expect that the optimal commitment period decreases with the size of the fiscal package. This is shown in table 4, where the welfare losses resulting from a policy mix are minimized with a commitment of 23 periods, i.e. 3 periods less than in the case of pure management of expectations without a fiscal stimulus.

7 Conclusion

This paper investigates the qualitatively impact of technology, preferences, and financial markets shocks that make the zero lower bound on the nominal interest rate binding. The shocks on the credit market we consider are a decrease in the survival probability of entrepreneurs (or a negative net worth shock), and an increase in the entrepreneurs' returns volatility (or a positive risk premium shock). We describe how the economy is subject to bigger fluctuations when the central bank cannot reduce further the nominal interest rate.

Considering a recession generated from a negative net worth shock and a shift from spending to saving by households, we explore economic policies that search to steer the economy back to the long run equilibrium. The economic measures we consider are a fiscal stimulus, in terms of an increase in government purchases, and the management of public expectations, in terms of the future path of the nominal interest rate, as recommended by Eggertsson and Woodford (2003).

Several results emerge. When the nominal interest rate hit its zero bound, the monetary authorities lost its policy instrument, resulting in a stronger volatility real and nominal variables. In addition, when an unconditional commitment is a pledge by the central bank to keep the interest rate at lower levels for more time than prescribed by a typical interest rate rule, the volatility of output and inflation is reduced. When it comes to the counter-cyclical fiscal policy, we find that the fiscal multiplier is greater than one for at least five quarters when the zero lower bound is binding. Lastly, we emphasize the favorable impact of a policy mix between fiscal policy and forward guidance is implemented, which results in a fiscal multiplier greater than one for a long period of time (i.e. almost two years). Indeed, the zero lower bound on the nominal interest rate increases the net effect of government spending as a counter-cyclical policy.

The importance of the banking intermediation sector has been spotlighted during the recent financial turmoil. In our model, we have focused on the role of entrepreneurs' balance sheet in transmission mechanisms of shocks, assuming that the financial intermediary is perfectly competitive. The next step of this paper would be to assess how imperfect competitive banks modify the transmission mechanisms of financial shocks that make the zero lower bound on the nominal interest rate binding. We leave this topic for future research.

References

[1] Adam, K., and R., Billi. 2007. Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates. Journal of Monetary Economics, 54, 728-752.

[2] Anderson, G., G., Moore, 1985. A linear algebraic procedure for solving linear perfect foresight models. Economics Letters, 17, 247-252.

[3] Auerbach, A., and W., Gale, 2009. Activist Fiscal Policy to Stabilize Economic Activity. NBER Working Paper, 15407.

[4] Bernanke, B., M. Gertler, and S. Gilchrist., 2000. The Financial Accelerator in a Quantitative Business Cycle Framework. In Handbook of Macroeconomics, 1, 1341–93.

[5] Blanchard, O., and M., Kahn, 1980. The Solution of Linear Difference Models under Rational Expectations. Econometrica, 48, 1305-1311.

[6] Bodenstein, M., C.J. Erceg, 2009. The effects of Foreign Shocks when Interest Rate are at zero. International Finance Discussion Paper of the Federal Reserve, 983.

[7] Calvo, G. A., 1983. Staggered prices in a utility-maximizing framework. Journal of Monetary Economics, 12, 383-398.

[8] Carlstrom, Charles T, and S., Fuerst, 1997. Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis. American Economic Review, 87, 893-910.

[9] Christensen I. and A. Dib, 2008. The Financial Accelerator in an Estimated New Keynesian Model. Review of Economic Dynamics, 11, 155-178.

[10] Christiano, L.J., R. Motto, and M. Rostagno, 2003. The Great Depression and the Friedman-Schwartz Hypothesis. Journal of Money Credit and Banking, 35, 1119-97.

[11] Christiano, L.J., M. Eichenbaum, and S. Rebelo, 2009a. When the Government Spending Multiplier Large? NBER Working Paper 15394.

[12] Christiano, L.J., R. Motto, and M. Rostagno, 2009b. Financial Factors in Economic Fluctuations. Manuscript.

[13] Coenen, G., Orphanides, A., and V., Wieland, 2004. Price Stability and Monetary Policy Effectiveness when Nominal Interest Rates are Bounded at Zero. The B.E. Journal of Macroeconomics, 0.

[14] Cogan, J.F., T., Cwik, J.B., Taylor and V., Wieland, 2009. New Keynesian versus Old Keynesian Government Spending Multipliers. NBER Working Papers 14782.

[15] Dieppe, A., and P., McAdam, 2006. Monetary Policy under a Liquidity Trap: Simulation Evidence for the Euro Area. Journal of the Japanese and International Economies, 20, 338-363.

[16] Eggertson, G., and M. Woodford, 2003. The Zero Bound in Interest Rates and Optimal Monetary Policy. Brokkings Papers on Economic Activity, 1, 139-233.

[17] Faia, E., 2007. Finance and International Business Cycles. Journal of Money, Credit and Banking, 39, 151-185.

[18] Fernandez-Villaverde, J., 2010. Fiscal Policy in a Model with Financial Frictions. American Economic Review, Papers and Proceding, Forthcoming.

[19] Fukunaga, 2002. Financial Accelerator Effects in Japan's Business Cycles. Bank of Japan, working paper 02-6.

[20] Gilchrist, S. and J., Leahy, 2002. Monetary Policy and Asset Prices. Journal of Monetary Economics, 49, 75-97.

[21] Levin, A., López-Salido, D., Nelson, E., and T., Yun, 2009. Limitations on the Effectiveness of Forward Guidance at the Zero Lower Bound. CEPR Discussion Paper, 7581.

[22] Meier A., and G., Müller, 2006. Fleshing out the Monetary Transmission Mechanism - Output Composition and the Role of Financial Frictions. Journal of Money, Credit, and Banking, 38, 2099-2133.

[23] Nakov, A., 2008. Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rate. International Journal of Central Banking, 4, 73-127.

[24] Nolan, C., and C. Thoenissen, 2009. Financial Shocks and the US Business Cycle. Journal of Monetary Economics, 56, 596-604.

[25] Oda, N. and N., Takashi, 2008. On the Function of the Zero Interest Rate Commitment: Monetary Policy Rules in the Presence of the Zero Lower Bound on Interest Rates. Journal of the Japanese and International Economies, 22, 34-67.

[26] Queijo von Heideken, V., 2009. How Important are Financial Frictions in the U.S. and the Euro Area? Scandinavian Journal of Economics, 111, 567-96.

[27] Reifschneider, D., and J. C., Williams, 2000. Three Lessons for Monetary Policy in a Low-Inflation Era. Journal of Money, Credit and Banking, 32, 936-966.

[28] Rudebusch, G., 2009. The Fed's Monetary Policy Response to the Current Crisis. FRBSF Economic Letter, 2009-17.

[29] Romer, C., and J., Bernstein, 2009. The Job Impact of the American Recovery and Reinvestment Plan. Manuscript

[30] Smets, F., and R. Wouters, 2007. Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. American Economic Review, 97, 586-606.

[31] Townsend, R. M., 1979. Optimal Contracts and Competitive Markets with Costly State Verification. Journal of Economic Theory 21, 265-93.

[32] Jung, T., T., Yuki and W., Tsutomu, 2005. Optimal Monetary Policy at the Zero-Interest-Rate Bound. Journal of Money, Credit, and Banking, 37, 813-835.

[33] Wolman, A., 2005. Real Implications of the Zero Bound on Nominal Interest Rates. Journal of Money, Credit and Banking, 37, 273-296.

8 Appendix

The log-linear model can be sum up as following:

8.0.1 Household's preferences

Euler equation

$$(1-\beta b)\,\sigma\hat{\lambda}_t = \beta b \mathrm{E}_t\{\hat{c}_{t+1}\} - (1+\beta b^2)\hat{c}_t + b\hat{c}_{t-1} + \sigma\hat{\varepsilon}_t - \beta b\sigma \mathrm{E}_t\{\hat{\varepsilon}_{t+1}\}, \tag{25}$$

where $\sigma^{-1} = -\mathbb{U}_{cc}c/\mathbb{U}_c$.

Risk free bond equation

$$\hat{\lambda}_t - \hat{R}_t = \mathrm{E}_t\left\{\hat{\lambda}_{t+1} - \hat{\pi}_{t+1}\right\} \qquad \text{and} \qquad \hat{r}_t = \hat{R}_t - \mathrm{E}_t\{\hat{\pi}_{t+1}\}. \tag{26}$$

Wage Phillips curve

$$\hat{\pi}^w_t - \gamma_w\hat{\pi}_{t-1} = \frac{(1-\alpha_w)(1-\beta\alpha_w)}{\alpha_w(1+\omega_w\theta_w)}\left[\omega_w\hat{\ell}^h_t - \hat{\lambda}_t - \hat{w}_t\right] + \beta \mathrm{E}_t\{\hat{\pi}^w_{t+1} - \gamma_w\hat{\pi}_t\}, \tag{27}$$

where $\omega_w = \ell^h \mathbb{V}_{\ell\ell}/\mathbb{V}_\ell$.

Wage inflation definition

$$\hat{\pi}_t^w = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t. \tag{28}$$

8.0.2 Intermediate Good Sector

Production function

$$\hat{y}_t = \hat{A}_t + (1-\alpha)\hat{\ell}_t + \alpha\hat{k}_t \qquad \text{and} \qquad \hat{\ell}_t = \Omega\hat{\ell}_t^h. \tag{29}$$

Cost minimization

$$\hat{w}_t = \hat{s}_t + \hat{y}_t - \hat{\ell}_t^h, \qquad \hat{w}_t^e = \hat{s}_t + \hat{y}_t, \qquad \text{and} \qquad \hat{z}_t = \hat{s}_t + \hat{y}_t - \hat{k}_t \tag{30}$$

Phillips curve

$$\hat{\pi}_t - \gamma_p\hat{\pi}_{t-1} = \frac{(1-\alpha_p)(1-\beta\alpha_p)}{\alpha_p}\hat{s}_t + \beta \mathrm{E}_t\left\{\hat{\pi}_{t+1} - \gamma_p\hat{\pi}_t\right\}, \tag{31}$$

8.0.3 Entrepreneur

$$\hat{x}_t = \hat{q}_{t-1} + \widehat{\tilde{k}}_t - \hat{n}_t, \qquad \text{and} \qquad \widehat{\check{r}}_t = \hat{r}_t^k - \hat{r}_{t-1} \tag{32}$$

Optimal contract equations

$$\mathrm{E}_t\{\widehat{\check{r}}_{t+1}\} + \mathrm{E}_t\{\hat{\sigma}_{t+1}\}\sigma\check{r}\left[\frac{\Gamma_\sigma(\bar{\omega})}{\Gamma_\omega(\bar{\omega})}\mu G_\omega(\bar{\omega}) - \mu G_\sigma(\bar{\omega})\right] = \mathrm{E}_t\{\hat{\Lambda}_{t+1}\}\left[1 - \check{r}\left[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right]\right], \tag{33}$$

$$\mathrm{E}_t\{\hat{\Lambda}_{t+1}\} = \mathrm{E}_t\{\widehat{\bar{\omega}}_{t+1}\}\bar{\omega}\left[\frac{\Gamma_{\omega\omega}(\bar{\omega})}{\Gamma_\omega(\bar{\omega})} - \frac{\Gamma_{\omega\omega}(\bar{\omega}) - \mu G_{\omega\omega}(\bar{\omega})}{\Gamma_\omega(\bar{\omega}) - \mu G_\omega(\bar{\omega})}\right] + \mathrm{E}_t\{\hat{\sigma}_{t+1}\}\sigma\left[\frac{\Gamma_{\omega\sigma}(\bar{\omega})}{\Gamma_\omega(\bar{\omega})} - \frac{\Gamma_{\omega\sigma}(\bar{\omega}) - \mu G_{\omega\sigma}(\bar{\omega})}{\Gamma_\omega(\bar{\omega}) - \mu G_\omega(\bar{\omega})}\right], \tag{34}$$

$$\hat{x}_{t+1} = \mathrm{E}_t\{\widehat{\check{r}}_{t+1}\}\left[x-1\right] + \mathrm{E}_t\{\widehat{\bar{\omega}}_{t+1}\}\bar{\omega}\check{r}x\left[\Gamma_\omega(\bar{\omega}) - \mu G_\omega(\bar{\omega})\right] + \mathrm{E}_t\{\hat{\sigma}_{t+1}\}\sigma\check{r}x\left[\Gamma_\sigma(\bar{\omega}) - \mu G_\sigma(\bar{\omega})\right]. \tag{35}$$

Return of capital

$$\mathrm{E}_t\{\hat{r}_{t+x1}^k\} = \mathrm{E}_t\{\hat{z}_{t+1}\}\frac{z}{r^k} + \mathrm{E}_t\{\hat{q}_{t+1}\}\frac{(1-\delta)}{r^k} - \hat{q}_t. \tag{36}$$

Net worth

$$\hat{n}_{t+1}\frac{1}{xr^k} = (\hat{\gamma}_t + \hat{v}_t)\gamma[1-\Gamma(\bar{\omega})] + \hat{w}_t^e\left[\frac{1}{r^k x} - \gamma[1-\Gamma(\bar{\omega})]\right], \tag{37}$$

$$\hat{v}_t[1-\Gamma(\bar{\omega})] = \hat{r}_t^k\left[1-\mu G(\bar{\omega})\right] + \hat{n}_t[1-\Gamma(\bar{\omega})] + \hat{x}_t\left[1 - \frac{1}{\check{r}} - \mu G(\bar{\omega})\right] - \hat{r}_{t-1}\frac{1}{\check{r}}\left[1 - \frac{1}{x}\right] - \widehat{\bar{\omega}}_t\bar{\omega}\mu G_\omega(\bar{\omega}) - \hat{\sigma}_t\sigma\mu G_\sigma(\bar{\omega}), \tag{38}$$

$$\hat{c}_t^e\frac{c^e}{\tilde{k}r^k} = (\hat{v}_t(1-\gamma) - \hat{\gamma}_t\gamma)\,\varrho[1-\Gamma(\bar{\omega})]. \tag{39}$$

8.0.4 Capital producer

Law of motion of capital

$$\widehat{\tilde{k}}_{t+1}\frac{1}{\delta} = \hat{\imath}_t + \widehat{\tilde{k}}_t\left[\frac{1}{\delta} - 1\right] - \hat{u}_t\frac{z}{\delta}. \tag{40}$$

Price of capital

$$\hat{q}_t\frac{1}{\varkappa} = \hat{\imath}_t[1+\beta] - \hat{\imath}_{t-1} - \beta\mathrm{E}_t\{\hat{\imath}_{t+1}\}, \tag{41}$$

where $\varkappa = \Phi''(1)$.

Capital utilization rate

$$\hat{k}_t = \hat{u}_t + \widehat{\tilde{k}}_t \quad \text{and} \quad \hat{z}_t = \vartheta_u\hat{u}_t + \hat{q}_t, \quad \text{where}\vartheta_u = u\delta''(u)/\delta'(u). \tag{42}$$

8.0.5 Resource constraint

$$\hat{y}_t = \hat{c}_t\frac{c}{y} + \hat{\imath}_t\frac{i}{y} + \hat{c}^e_t\frac{c^e}{y} + \hat{g}_t\frac{g}{y} + \left[\hat{r}^k_t + \hat{q}_{t-1} + \widehat{\tilde{k}}_t\right]\left[\mu G(\bar{\omega})r^k\frac{\tilde{k}}{y}\right] + \widehat{\bar{\omega}}_t\bar{\omega}\mu G_\omega(\bar{\omega})r^k\frac{\tilde{k}}{y} + \hat{\sigma}_t\sigma\mu G_\sigma(\bar{\omega})r^k\frac{\tilde{k}}{y} + \hat{\mu}_t\mu G(\bar{\omega})r^k\frac{\tilde{k}}{y}. \tag{43}$$

8.0.6 Non-constraint monetary policy

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)a_\pi\hat{\pi}_t + (1-\rho_R)a_y\hat{y}_t + (1-\rho_R)a_q\hat{q}_t \tag{44}$$

Table 1. Calibrated Parameters

Preferences and Technology		Value
β	Discount factor	0.99
b	Degree of habit consumption	0.63
σ	Inv. of the elasticity of intertemp. substitution	0.20
ω_w	Elasticity of labor disutility	1.00
ϕ	Elasticity of value added wrt capital	0.33
δ	Capital depreciation rate	0.03
$\varkappa$	Investment adjustment cost	5.86
ϑ_u	Utilization rate of capital parameter	1/0.31
Nominal Rigidities		
θ_p	Elasticity of substitution of goods	11.00
α_p	Degree of price stickiness	0.75
γ_p	Degree of price indexation	0.75
θ_w	Elasticity of substitution of labor	21.00
α_w	Degree of wage stickiness	0.80
γ_w	Degree of wage indexation	0.70
Fiscal and Monetary Policy		
g/y	Share of government expenditure in ouput	0.22
ρ_R	Interest rate smoothing	0.88
a_p	Elasticity of the interest rate wrt inflation	1.88
a_y	Elasticity of the interest rate wrt output gap	0.11
a_q	Elasticity of the interest rate wrt return on capital	0.00

Table 2. Calibrated Parameters

Financial Accelerator Mechanism		Value
Ω	Share of household labor in aggr. labor	0.9846
x	Steady-state ratio of capital to net worth	2.00
$\check{r}$	Steady-state risk spread	$1.02^{0.25}$
γ	Survival rate of entrepreneurs	0.9788
$\bar{\omega}$	Threshold value of idiosyncratic shock	0.4982
σ_ω	Standard error of idiosyncratic shock	0.2764
μ	Monitoring cost	0.1175

Table 3. Values of the loss function for different interest rate paths

Interest rate is equal to ZLB for...	Loss function value
16 periods	0.08
20 periods	0.06
25 periods	0.01
30 periods	0.05

Table 4. Welfare losses resulting from the policy mix

Interest rate is equal to ZLB for...	Loss function value
15 periods	0.04
20 periods	0.02
23 periods	0.004
25 periods	0.01
30 periods	0.10

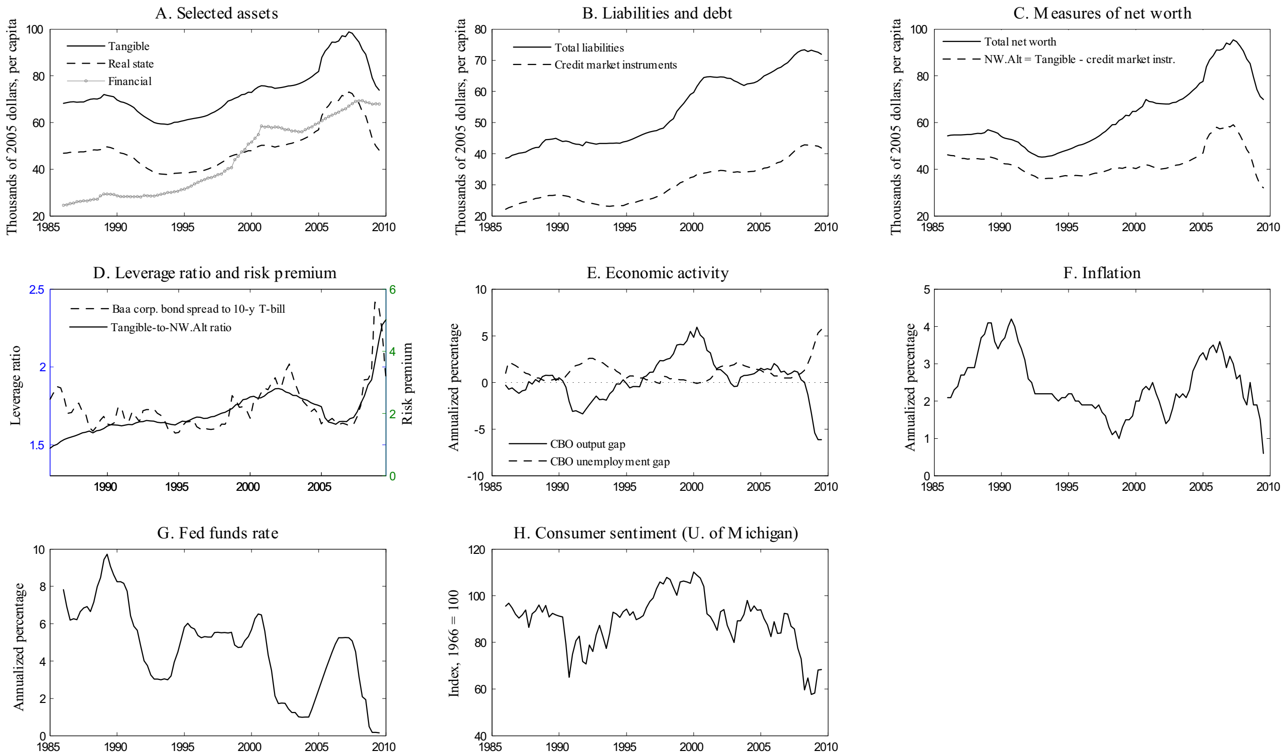

Figure 1: Economic and Nonfarm Nonfinancial Corporate Business Data

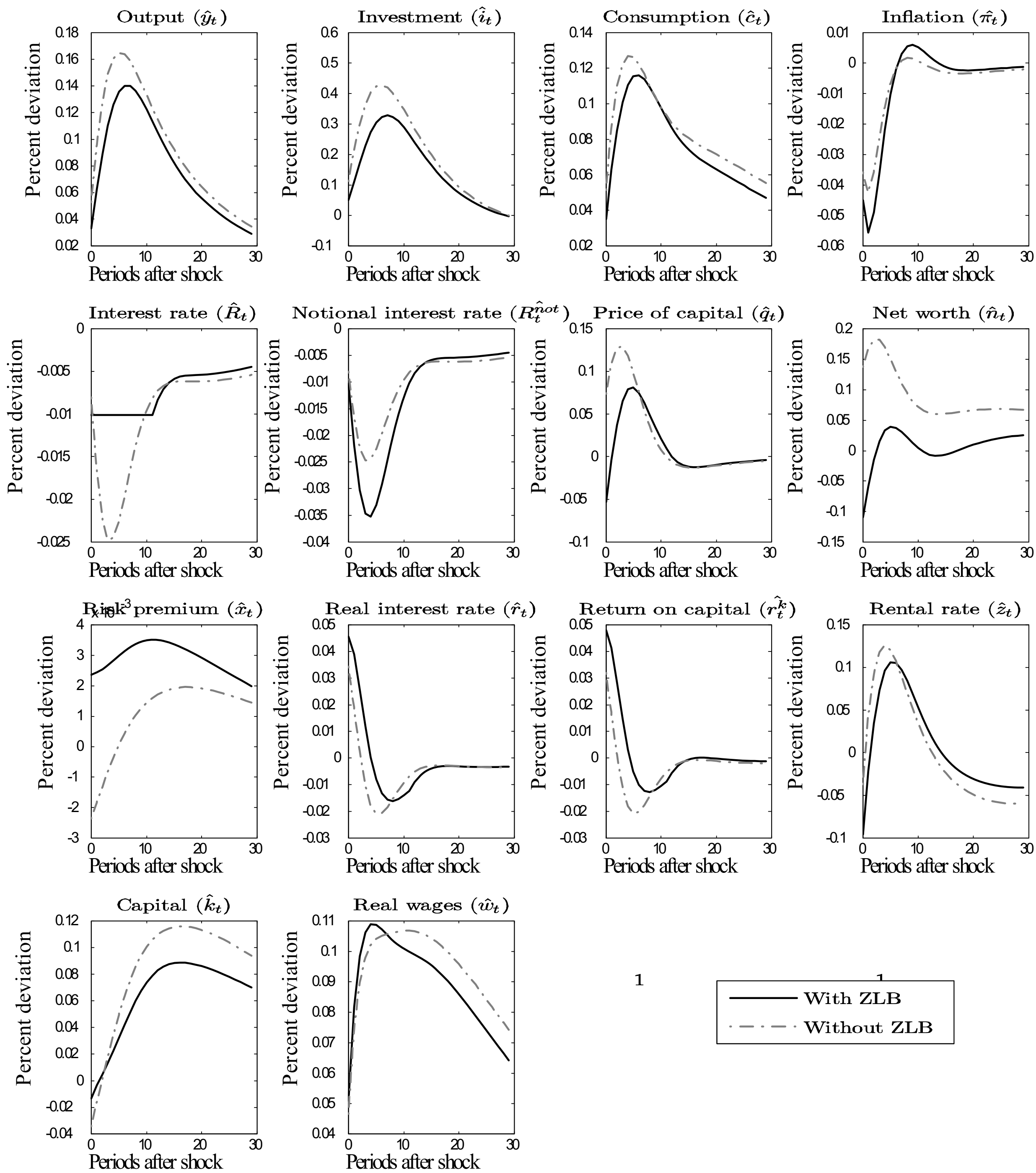

Figure 2: IRFs to a technology shock

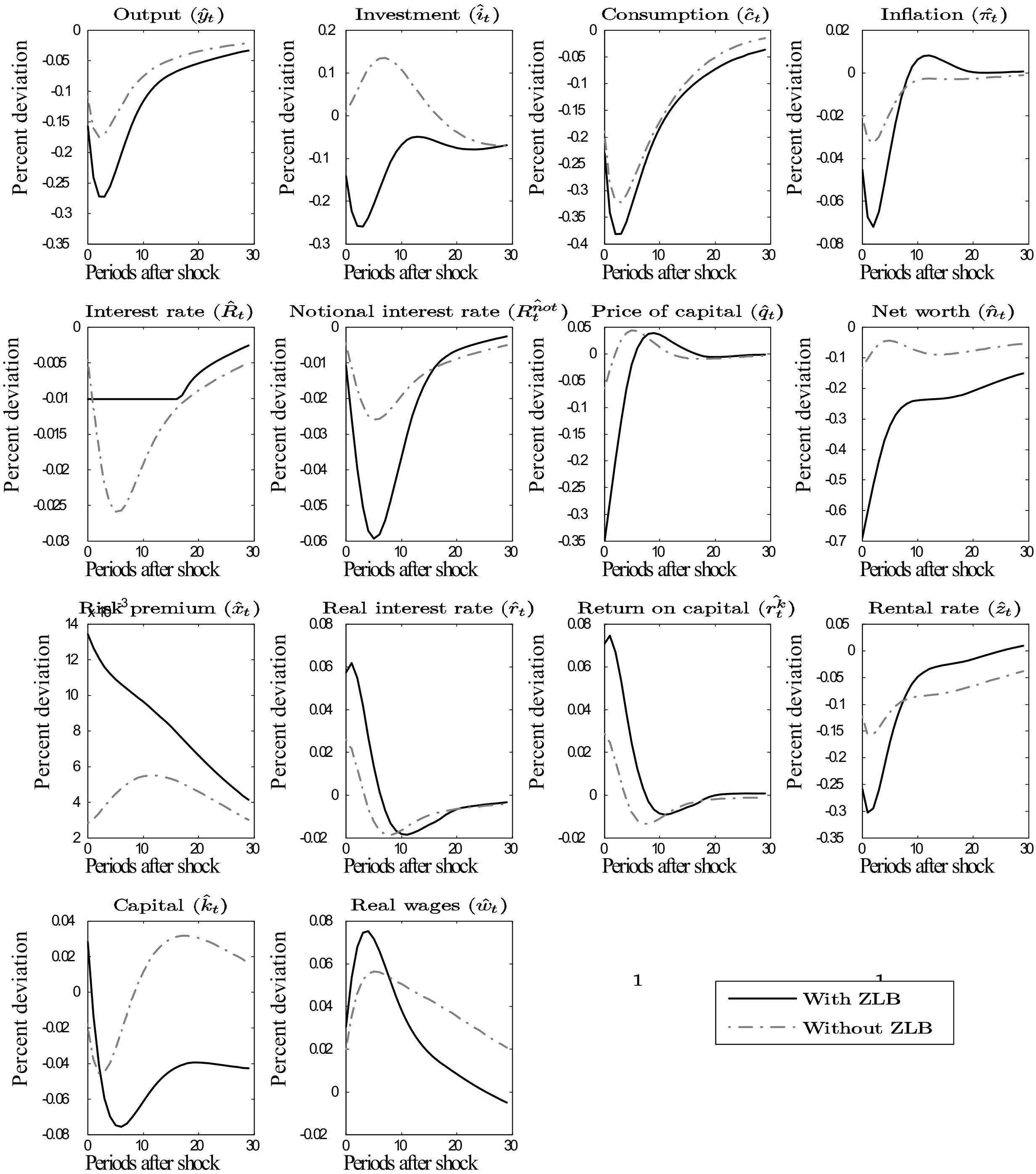

Figure 3: IRFs to a preference shock

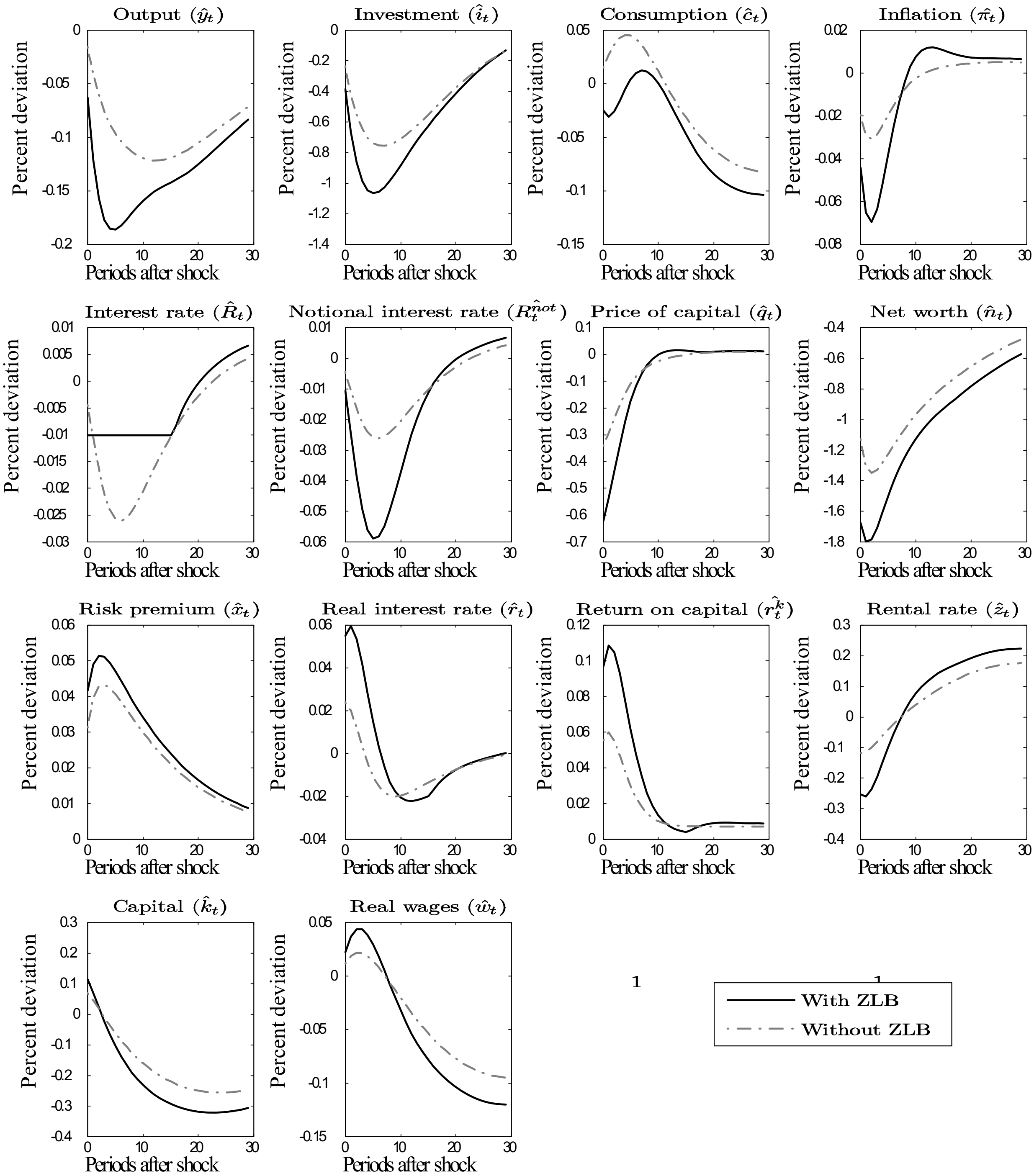

Figure 4: IRFs to a net worth shock

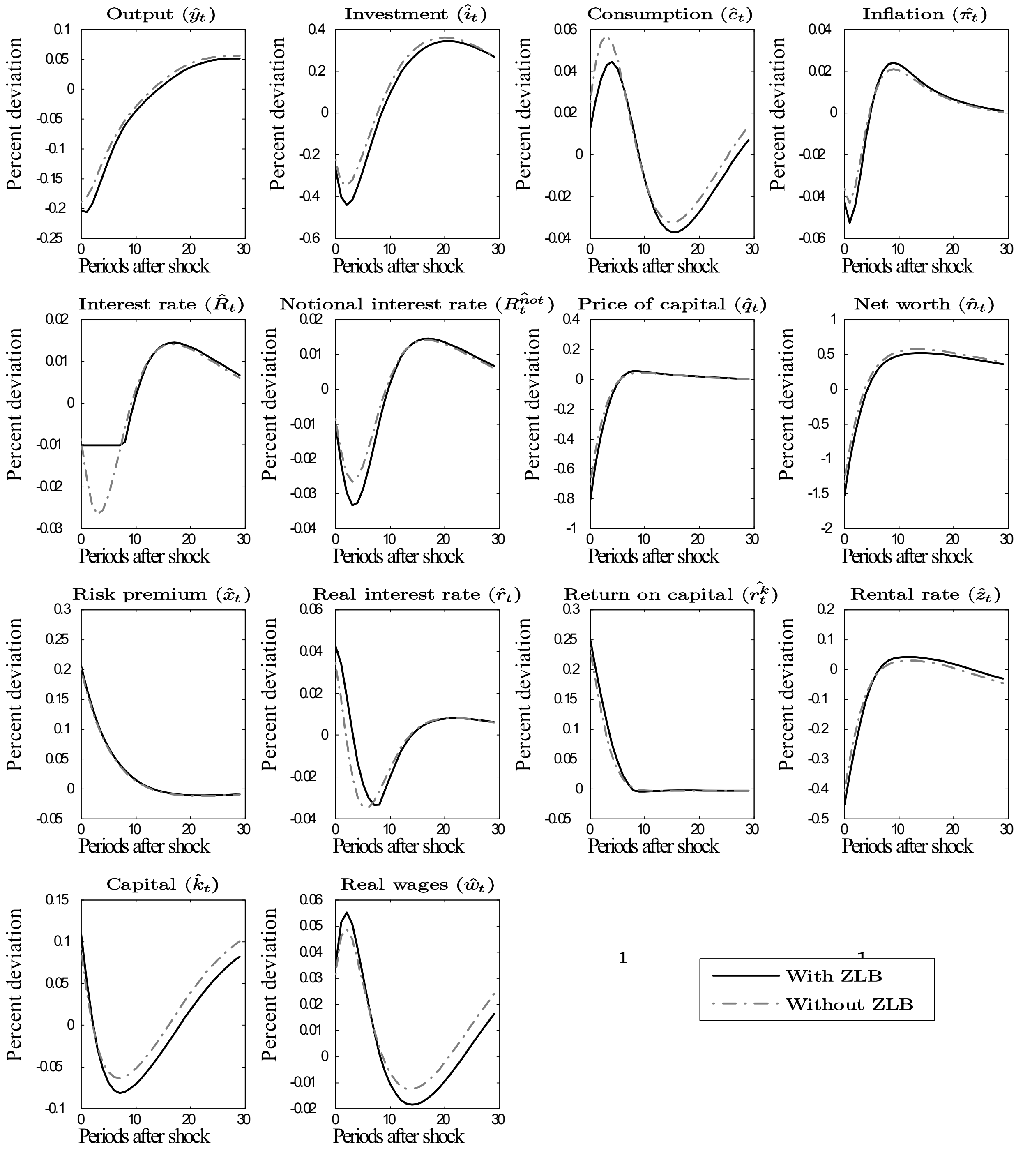

Figure 5: IRFs to a risk shock, with $\rho_\omega = 0.85$

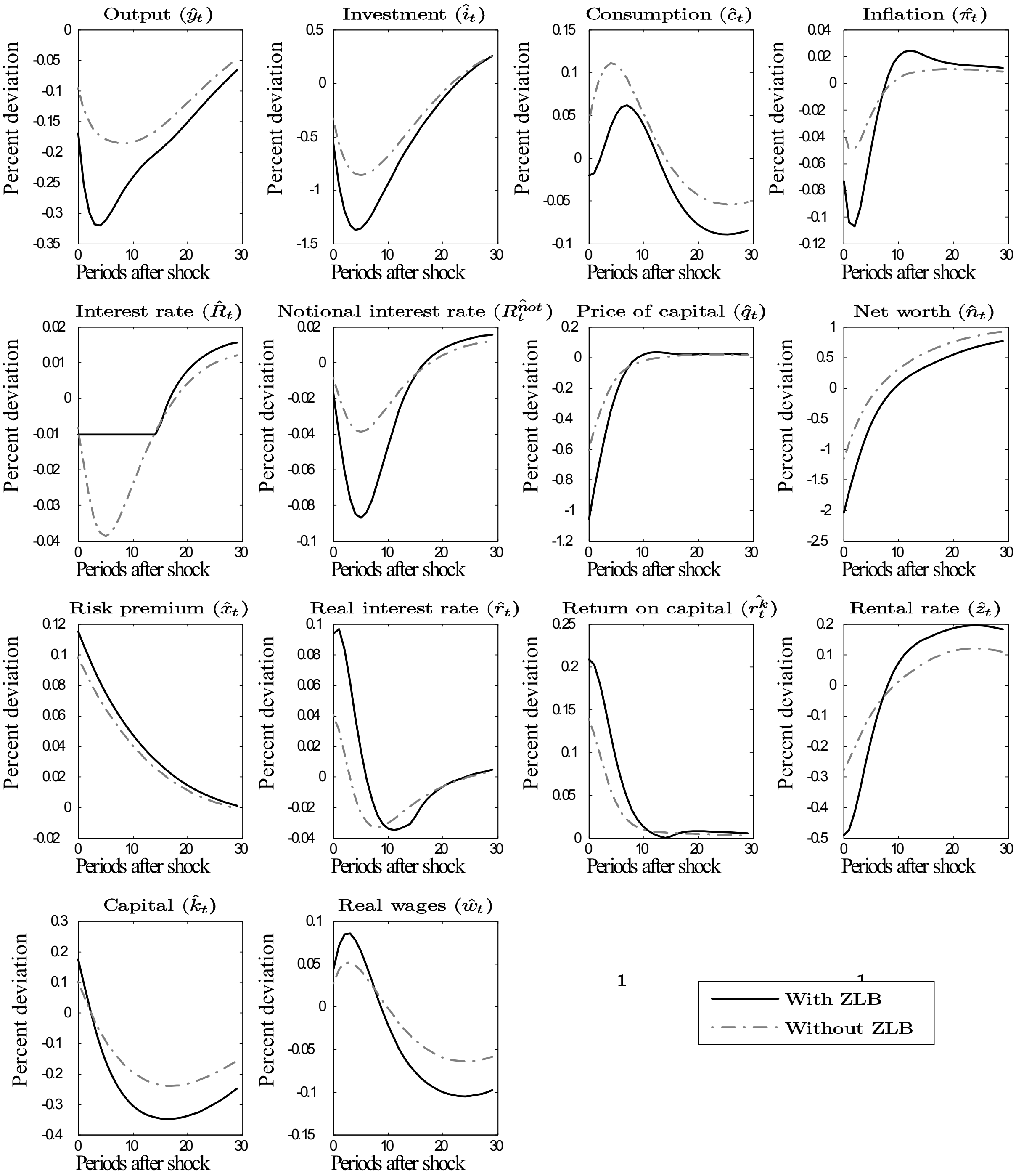

Figure 6: IRFs to a risk shock, with $\rho_\omega = 0.98$

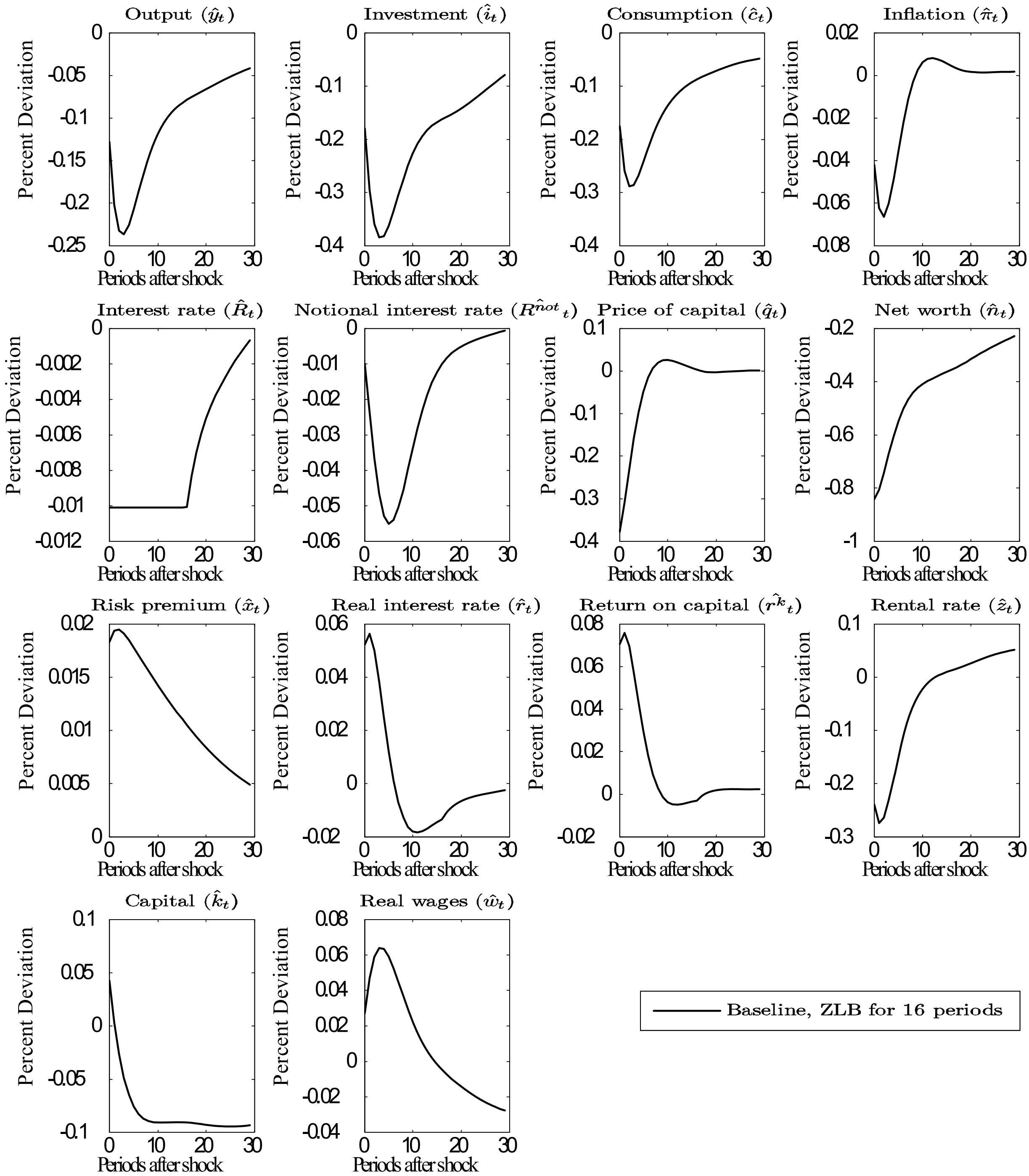

Figure 7: IRFs of a deep recession. Total effects of a preference and net worth shocks.

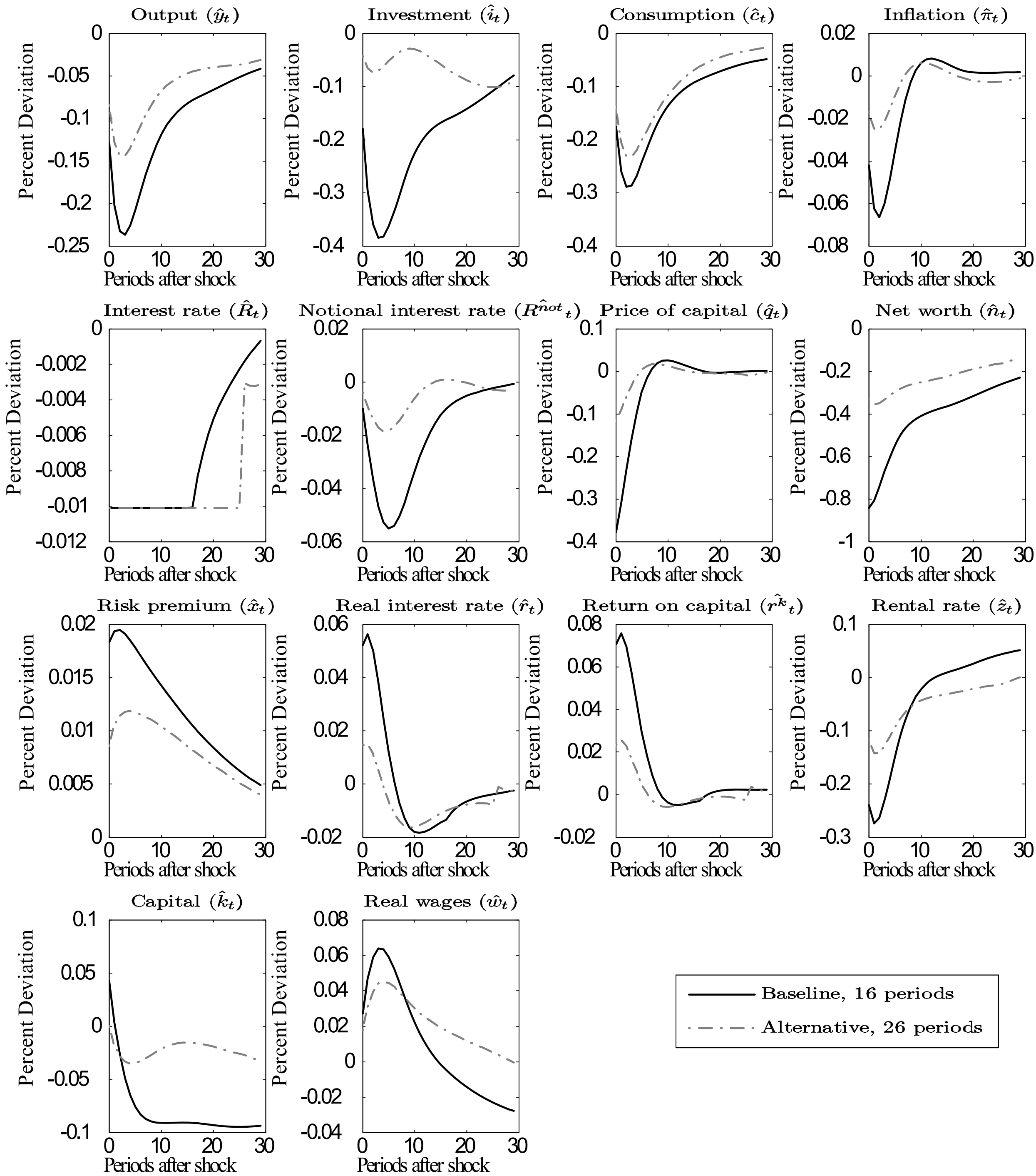

Figure 8: Deep economic recession and managing interest rate expectations.

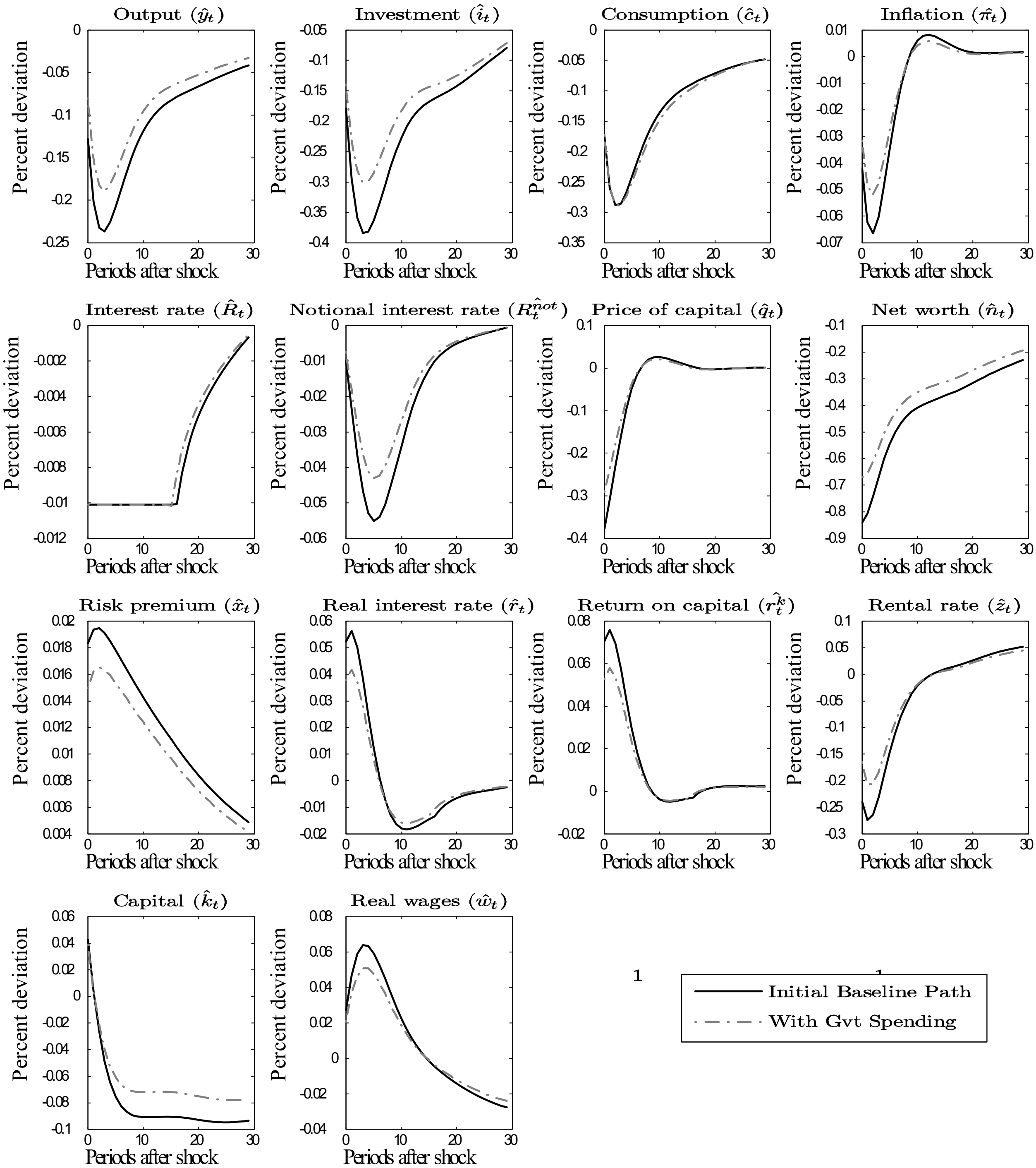

Figure 9: Deep economic recession and increase of government purchases.

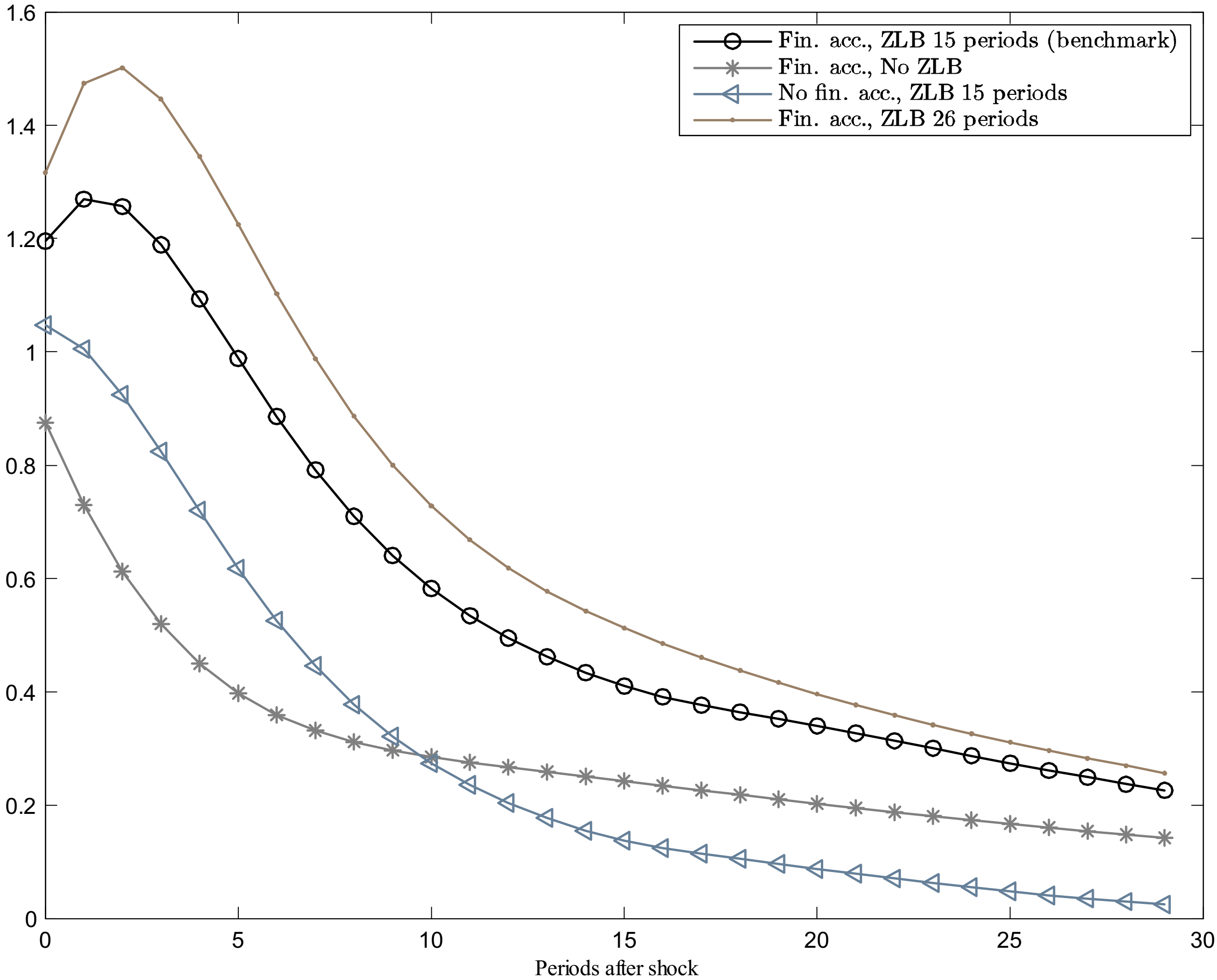

Figure 10: Fiscal multiplier for different configurations